Optimisation in Multiple View Geometry: The L-infinity Way

III - Large-Scale Methods

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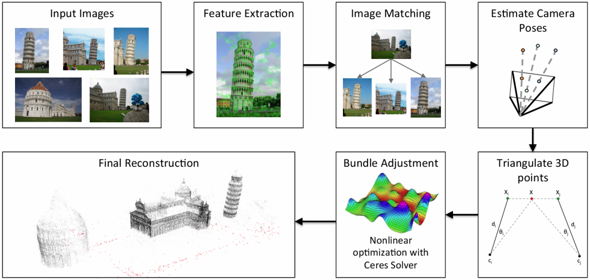
CVPR 2018 Tutorial

Overview

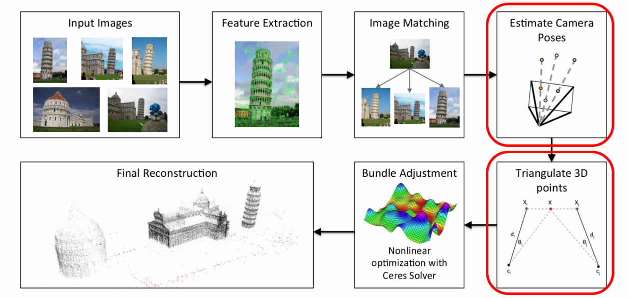
1. L_∞ and the Structure from Motion Pipeline
   - Rotation Averaging
   - L_∞ Known Rotation Problem
2. Pseudo-Convexity
   - Quasi-Convexity recap
   - Other types of Convexity
   - L_∞ and Pseudoconvexity
3. Large-Scale Methods for L_∞
   - Interior-Point Methods
   - Proximal Splitting Methods
   - Resection-Intersection

Structure from Motion Pipeline



Typical Structure-from-Motion Pipeline.

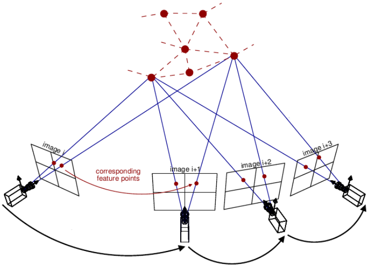
Structure from Motion Pipeline



The L_∞ formulation can solve core subproblems in the Structure-from-Motion (SfM) pipeline.

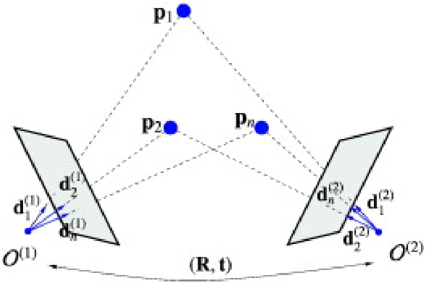
- Estimate Camera Pose
- Estimate 3D points
- Rotation Averaging + Known Rotation Problem

Estimate Camera Orientation



1. Estimate relative (pairwise) rotation (Epipolar Geometry)
2. Find absolute rotations (Rotation Averaging)
3. Solve the Known Rotation Problem

Estimating Relative Rotation



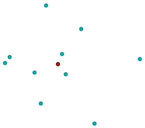
- Use epipolar geometry estimate relative rotations between all camera pairs

Rotation Averaging

- Centroid:

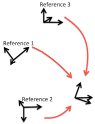
$$\min_{x} \sum_{i=1}^{N} ||\hat{x}_i - x||_2^2$$

mean (or average) of the points



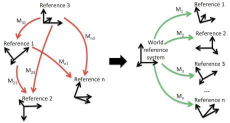
- (Single) Rotation Averaging

$$\min_{R \in SO(3)} \sum_{i=1}^{N} ||\hat{R}_i - R||^2$$



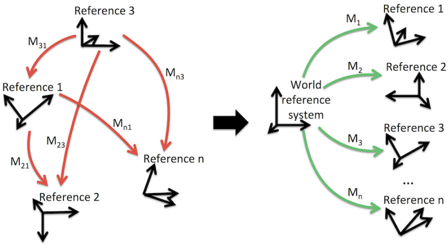
- (Multiple) Rotation Averaging

$$\min_{R_i \in SO(3)} \sum_{ij=1}^{N} ||R_i \hat{R}_{ij} - R_j||^2$$



(Also known as *Rotation Synchronization*)

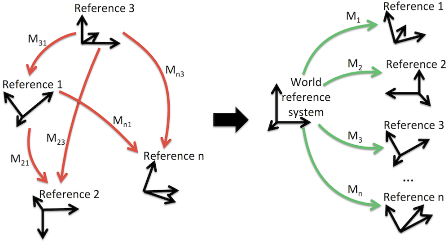
Rotation Averaging



Determine absolute camera rotations from pairwise relative measurements.

$$R_i R_{ij} = R_j, \quad \forall (i,j) \in \mathcal{N}$$

Rotation Averaging

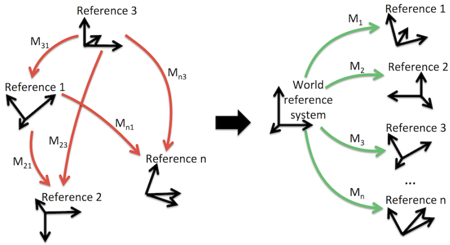


Determine absolute camera rotations from pairwise relative measurements.

$$\underset{R_1,\dots,R_n}{\arg\min} \sum_{(i,j)\in\mathcal{N}} d(R_i\tilde{R}_{ij}, R_j)^p,$$

where $p \geq 1$ and $d(\cdot,\cdot)$ is a distance function.

Rotation Averaging

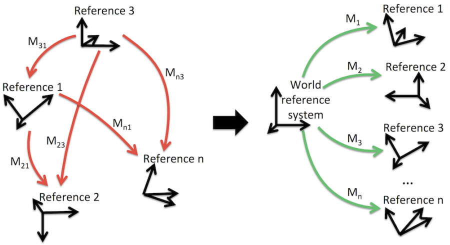


Determine absolute camera rotations from pairwise relative measurements.

$$(P) \quad \underset{R_i \in SO(3)}{\arg\min} \sum_{(i,j) \in \mathcal{N}} ||R_i \tilde{R}_{ij} - R_j||_F^2, \qquad \textit{(chordal distance)}$$

(P) is a non-convex problem.

Rotation Averaging



A number of methods exist for solving this problem:

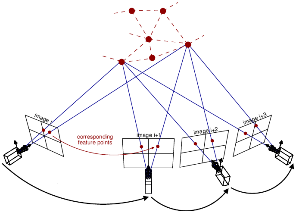
- efficiently [8]
- robustly [6]
- globally [1]*

*Rotation Averaging and Strong Duality

Anders Eriksson, Carl Olsson, Fredrik Kahl and Tat-Jun Chin. CVPR 2018.

Oral Session Tues 8:50am Room 255

L_∞ Known Rotation Problem



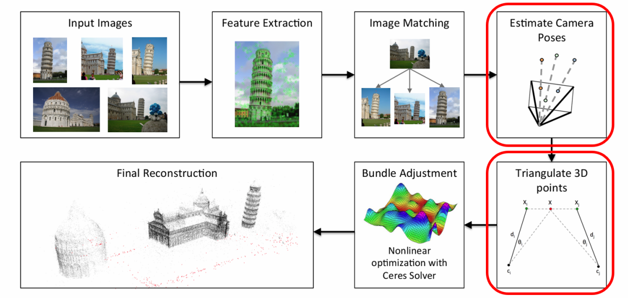
Solve:

$$\min_{\{t,x\}\in\mathcal{C}} \quad \max_{j,k} \left\| u_{j,k} - \frac{R_j^{1:2}x_k + t_j^{1:2}}{R_j^3 x_k + t_j^3} \right\|_p \quad \text{(KRot)}$$

with $\mathcal{C} = \left\{ x, t \mid R_j^3 x_k + t_j^3 > 0, \ \forall j, k \right\}$

- Directly solvable in using L_∞ formulation
- Potentially a large-scale problem, (no. cameras and 3D-points)

Structure from Motion Pipeline



The L_∞ formulation can solve core subproblems in the Structure-from-Motion (SfM) pipeline.

- Rotation Averaging
- L_∞ Known Rotation Problem, $\mathbb{R}^{3(m+n)}$ (m:cams and n:3D-pts)

How to solve this potentially Large-Scale problems in practise?

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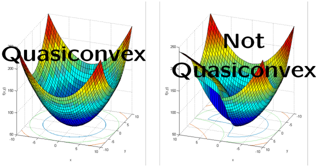
   - Quasi-Convexity recap
   - Other types of Convexity
   - L_∞ and Pseudoconvexity

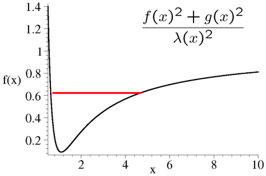
3. Large-Scale Methods for L_∞
   - Interior-Point Methods
   - Proximal Splitting Methods
   - Resection-Intersection

Quasi-Convexity - recap

Quasiconvex Function
A function f is *quasiconvex* if its sub-level sets $S_\alpha(f) = \{x \mid f(x) \leq \alpha\}$ are convex for all α.



Geometric Projection Error
The Geometric Projection Error is a quasiconvex function.

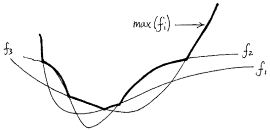


Quasi-Convexity - recap

- The sum of quasiconvex functions is not necessarily a convex or quasiconvex function.

- The pointwise maximum of quasiconvex functions is a quasiconvex function.

Proof:
If $f(x) = \max_i f_i(x)$ then $S_\alpha(f) = \bigcap_i S_\alpha(f_i)$.



Intersection of convex sets is convex.

Quasi-Convexity - recap

- Instead of minimizing the sum of the squared reprojection error

$$\min_X \sum_{i=1}^{N} \frac{f_i(X)^2 + g_i(X)^2}{\lambda_i(X)^2}$$

- minimize the largest reprojection error

$$\min_X \max_i \left\| \frac{[f_i(X), g_i(X)]}{\lambda_i(X)} \right\|_p$$

- also written as

$$\min_X \left\| \frac{[f_i(X), g_i(X)]}{\lambda_i(X)} \right\|_{p,\infty}$$

Quasi-Convexity - recap

Solve:

$$\min_x \max_i f_i(x)$$

Alternatively:

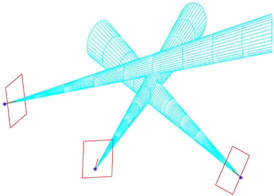
$$\begin{aligned} \min_{x,s} \quad & s \\ \text{s.t.} \quad & f_i(x) \leq s, \quad \forall i \end{aligned}$$

- A convex problem for fixed s.
- Find the smallest s such that the intersection $f_i(x) \leq s$ is not empty.

Quasi-Convexity - recap

Solve:

$$\min_{x,s} \quad s$$
$$\text{s.t.} \quad f_i(x) \leq s, \quad \forall i$$

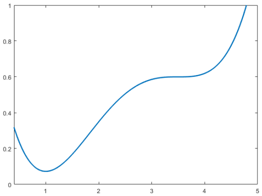


- A convex problem for fixed s.
- Find the smallest s such that the intersection is not empty.
- Solve using i.e. bisection (more clever algorithms for updating s can be applied, Brent's method, Dinkelbach's algorithm, Gugat's algorithm)

+ Root-Finding is guaranteed to find the global minima.
- Need to solve a feasibility problem at each iteration.

Quasi-Convexity

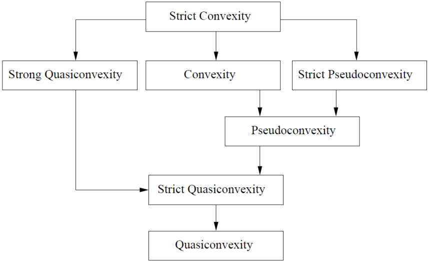
$$\min_{x,s} \quad s$$
$$\text{s.t.} \quad f_i(x) \leq s, \quad \forall i$$



A quasiconvex with several stationary points.

- \+ Guaranteed to find global minima.
- \- Need to solve a feasibility problem at each iteration.
- \- Slow
- \- Can have several local minima and saddlepoints ($\nabla f = 0$)
  $\Rightarrow$ Using standard optimization algorithms will not guarantee global optimality

Other types of Convexity



Definition (Pseudo-Convexity)

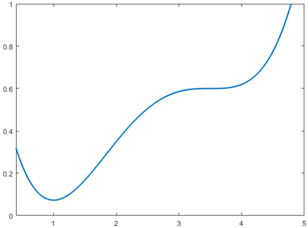
f is called pseudoconvex if f is differentiable and whenever $\nabla f(\bar{x})(x-\bar{x}) \geq 0$ we also have that $f(x) \geq f(\bar{x})$.

(Pseudoconvex $\Rightarrow$ Quasiconvex)

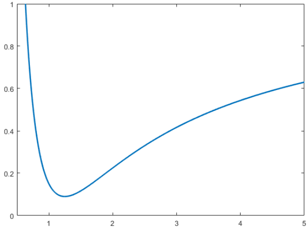
Lemma

If f is pseudoconvex, then $\nabla f(\bar{x}) = 0$ if and only if $f(x) \geq f(\bar{x})$ for all x.

Quasiconvex vs Pseudoconvex

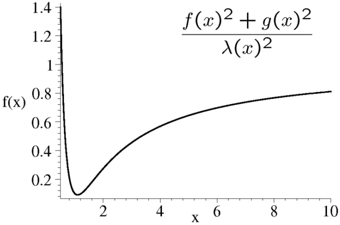


A quasiconvex (but not pseudoconvex) function
- Can not be solved using a local solver



A pseudoconvex function.
+ Can be solved using a local solver

L_∞ and Pseudoconvexity



In [13] it was shown that

$$f(x) = \frac{\left\| [a_1^T x + b_1, a_2^T x + b_2] \right\|^2}{(a_3^T x + b_3)^2}$$

is in fact a pseudoconvex function on $S = \{x | a_3^T x + b_3 > 0\}$.

$\max_i$ of Pseudoconvex Functions

Theorem (see [13])

If $f_i(x)$ are pseudoconvex functions then a stationary (or KKT) point of

$$\min_{X} \ \|f_i(X)\|_\infty \qquad \text{or} \qquad \begin{aligned} \min_{x,s} \quad & s \\ \text{s.t.} \quad & f_i(x) \le s, \quad \forall i \end{aligned}$$

will also be a global minima.

Opens up L_∞ problems to a whole world of non-linear solvers.

Solving L_∞ using Local Methods

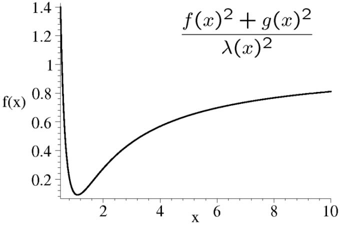
LOQO an interior point algorithm for general non-convex problems.

	bisection	*LOQO*
Triangulation:		
5 cameras	1.23	.00281
10 cameras	1.38	.00358
20 cameras	1.29	.00645
30 cameras	1.36	.00969
Homography:		
10 points	1.05	.00816
20 points	1.17	.0128
30 points	1.22	.0193
Resectioning:		
10 points	.823	.0128
20 points	.994	.0287
30 points	1.04	.0418

Average execution times (s).

- LOQO 50-100 times faster than bisection on smaller problems

Pseudo-Convexity - Summary



- L_∞ problems are **pseudoconvex**.
- Stationary points of pseudoconvex functions are **global minima**.
- **Local methods** can be used to solve our L_∞-problems.

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Three Algorithms for Large-Scale L_∞ Optimization

1. Interior-Point method
2. Pseudo-Convex Proximal Splitting
3. Block Coordinate Descent

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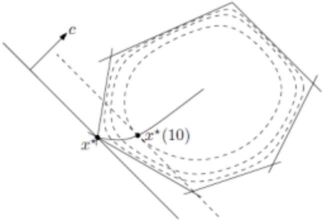
1 - Interior-Point Methods



$$\min_{x} \quad f(x)$$
$$\text{s.t.} \quad c_i(x) \le 0,$$
$$i \in 1, \ldots, m$$

- A class of algorithms for non-linear constrained optimization problems
- A standard formulation using barrier functions and Newton's algorithm

1 - Interior-Point Methods

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & c_i(x) \le 0, \\ & i \in 1, \dots, m \end{aligned}$$

Recall our L_∞-problem:

$$\begin{aligned} \min_x \quad & s \\ \text{s.t.} \quad & f_i(x) - s \le 0, \\ & i \in 1, \dots, m \end{aligned}$$

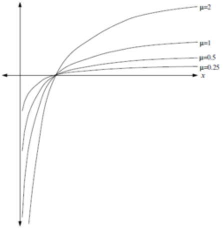
1 - Interior-Point Methods cont.

$$
\begin{aligned}
\min_{x} \quad & f(x) \\
\text{s.t.} \quad & c_i(x) - w = 0, \\
& w \geq 0, \\
& i \in 1, \ldots, m
\end{aligned}
$$

1 - Interior-Point Methods

$$\min_{x} \quad f(x) - \mu \sum_{i=1}^{m} \log w$$
$$\text{s.t.} \quad c_i(x) - w = 0,$$
$$i \in 1, \ldots, m$$

Let $\mu \to 0$



$\mu \log(w)$

1 - Interior-Point Methods

$$\min_{x} \quad f(x) - \mu \sum_{i=1}^{m} \log w + \lambda^T (c_i(x) - w)$$

λ Lagrangian multipliers

1 - Interior-Point Methods

$$\min_{x} \quad f(x) - \mu \sum_{i=1}^{m} \log w + \lambda^T (c_i(x) - w)$$

λ Lagrangian multipliers

Set gradient to 0

$$\begin{aligned} \nabla f(x) - \nabla c(x) &= 0 \\ -\mu W^{-1} e + \lambda &= 0 \\ c(x) - w &= 0 \end{aligned}$$

1 - Interior-Point Methods

Set gradient to 0

$$\begin{aligned} \nabla f(x) - \nabla c(x) &= 0 \\ -\mu W^{-1} e + \lambda &= 0 \\ c(x) - w &= 0 \end{aligned}$$

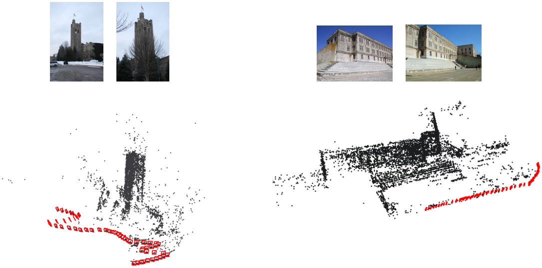
Solve using Newton's method

- A local method
- Well-studied and understood method
- LOQO has numerical issues for large problems
- Specialized Interior Point Methods for L_∞ problems has been proposed [3]
- Faster than bisection

QUT

Overview

2 - Proximal Splitting Methods [4]



- Approach based on Proximal Splitting methods (ADMM)
- Very simple to implement
- Efficient for Large-Scale problems

Alternating Direction Method of Multipliers - (ADMM)

Solve:

$$\begin{aligned} \min \quad & f(x) + g(z) \\ \text{s.t.} \quad & Ax + Bz = c \end{aligned}$$

Augmented Lagrangian:

$$L_\rho(x, z, y) = f(x) + g(z) + y^T(Ax + Bz - c) + (\rho/2)||Ax + Bz - c||_2^2$$

ADMM:

$$\begin{aligned} x_{k+1} &= \arg\min_x \; L_\rho(x, z_k, , y_k) && \text{// x-minimization} \\ z_{k+1} &= \arg\min_z \; L_\rho(x_{k+1}, z, y_k) && \text{// z-minimization} \\ y_{k+1} &= y_k + \rho(Ax_{k+1} + Bz_{k+1} - c) && \text{// dual update} \end{aligned}$$

Known Rotation Problem

Solve:

$$\min_{\{t,x\}\in\mathcal{C}} \quad \max_{j,k} \left\| u_{j,k} - \underbrace{\frac{R_j^{1:2}x_k + t_j^{1:2}}{R_j^3 x_k + t_j^3}}_{\Pi_j(x_k)} \right\|_p \qquad \text{(KRot)}$$

with $\mathcal{C} = \left\{ x, t \,|R_j^3 x_k + t_j^3 > 0, \;\; \forall j, k \right\}$

More Compact:

$$\min_{\{t,x\}\in\mathcal{C}} \quad \left\| u_{j,k} - \Pi_j(x_k) \right\|_{p,\infty} \qquad \text{(KRot)}$$

$$\left(Note: \;\; \min_{\{t,x\}\in\mathcal{C}} \quad \left\| u_{j,k} - \Pi_j(x_k) \right\|_2^2 \quad (L_2 \rightarrow \text{Bundle Adjustment}) \right)$$

Known Rotation Problem - cont.

$$\begin{aligned} \min_{\{t,x\}\in\mathcal{C}} \quad & \|Z\|_{p,\infty} \\ \text{s.t.} \quad & Z_{j,k} = u_{j,k} - \Pi_j(x_k) \end{aligned} \qquad \text{(KRot)}$$

Lagrangian:

$$L(x,Z,y) = \|Z\|_{p,\infty} + y^T(u - Z - \Pi(x)) + \frac{\rho}{2}\|u - Z - \Pi(x)\|_2^2 =$$

$$= \|Z\|_{p,\infty} + \frac{\rho}{2}\left\|\frac{1}{\rho}y + u - Z - \Pi(x)\right\|_2^2$$

Known Rotation Problem - cont.

Lagrangian:

$$L(x,Z,y) = \|Z\|_{p,\infty} + \frac{\rho}{2}\left\|\left(\frac{1}{\rho}y + u - Z\right) - \Pi(x)\right\|_2^2$$

Apply ADMM:

$$\begin{cases} x_{k+1} = \arg\min_x \; L_\rho(x, Z_k, y_k) = \arg\min_x \; \left\|\overbrace{\left(\frac{1}{\rho}y_k + u - Z_k\right)}^{\hat{u}_k} - \Pi(x)\right\|_2^2 = \\ \qquad = \arg\min_x \; \left\|\hat{u}_k - \Pi(x)\right\|_2^2 \quad \text{// x-minimization} \\ Z_{k+1} = \arg\min_Z \; L_\rho(x_{k+1}, Z, y_k) = \|Z\|_{p,\infty} + \frac{\rho}{2}\left\|\overbrace{\left(\frac{1}{\rho}y_k + u - \Pi(x_{k+1})\right)}^{\hat{w}_k} - Z\right\|_2^2 = \\ \qquad = \|Z\|_{p,\infty} + \frac{\rho}{2}\left\|\hat{w}_k - Z\right\|_2^2 \quad \text{// Z-minimization} \\ y_{k+1} = y_k + \rho(u - Z_{k+1} - \Pi(x_{k+1})) \quad \text{// dual update} \end{cases}$$

Subproblem in x

Solve:

$$x_{k+1} = \arg\min_x \left\| \hat{u}_k - \Pi(x) \right\|_2^2 \quad \text{// x-minimization}$$

$$\hat{u}_k = \left(\frac{1}{\rho} y_k + u - Z_k \right)$$

- Least squares minimization.
- Solve using Bundle Adjustment ...
- ... but now with respect to the modified image points $\hat{u}_k$.
- Not necessary to solve exactly.

Subproblem in Z

To solve the subproblem in Z we will solve problems on the form

$$Z_{k+1} = \|Z\|_{p,\infty} + \frac{\rho}{2}\left\|\hat{w}_k - Z\right\|_2^2 \quad \text{// Z-minimization}$$
$$\hat{w}_k = \frac{1}{\rho}y_k + u - \Pi(x_{k+1})$$

A convex problem in $Z \Rightarrow$ solve the problem in the dual

$$\underset{S}{\operatorname{argmax}} \quad -\frac{1}{2}||S||_F^2 + < S, w_k >,$$
$$\text{s.t.} \quad ||S||_{q,1} \leq \frac{1}{\rho}.$$

where $||\cdot||_q$ is the dual norm of $||\cdot||_p$, i.e. $\frac{1}{p} + \frac{1}{q} = 1$.

Subproblem in Z - cont.

// Z-minimization ($Z_{k+1} = w_k - S^*$)

$$S^* = \arg\min_S \quad \frac{1}{2}||S - w_k||_F^2$$
$$\text{s.t.} \quad ||S||_{q,1} \leq \frac{1}{\rho}$$

The related (regularized) problem

$$S_\theta^* = \arg\min_S \frac{1}{2}||S - w_k||_F^2 + \theta(||S||_{q,1} - \frac{1}{\rho}) = \arg\min_S \frac{1}{2}||S - w_k||_F^2 + \theta||S||_{q,1}$$

- We know that there must exist a θ such that $S_\theta^* \leq \frac{1}{\rho}$ and hence $S^* = S_\theta^*$
- How do we find it?

Turns out that S_θ^* has a very simple closed form solution...

Subproblem in Z - cont.

It can be shown that for $p = 2$ the solution is obtained by

$$S_\theta^* = \max(1 - \frac{\theta}{||w_k||_F}, 0) \odot w_k$$

A similar expressions also exists for $p = 1$, and $p = \infty$

- θ a scalar
- Find the θ for which $S_\theta^* = \frac{1}{\rho}$
  $\Leftrightarrow$ find a root of $g(\theta) = S_\theta^* - \frac{1}{\rho}$
- Very simple and efficient root-finding algorithm exists
- Solution is then given by $(Z_{k+1} = w_k - S_\theta^*)$

Pseudo-Convex Proximal Splitting

Input: u, R

Initialize: $x^0, y^0, Z^0, k = 0$

Repeat

- $\hat{u}_k = \left(\frac{1}{\rho} y_k + u - Z_k\right)$
- $x_{k+1} = \arg\min_x \left\| \hat{u}_k - \Pi(x) \right\|_2^2$ // x-update (n steps of BA)
- $\hat{w}_k = \frac{1}{\rho} y_k + u - \Pi(x_{k+1})$
- $Z_{k+1} = \|Z\|_{p,\infty} + \frac{\rho}{2} \left\| \hat{w}_k - Z \right\|_2^2$ // Z-update (solve $S_\theta - \frac{1}{\rho} = 0$)
- $y_{k+1} = y_k + \rho(u - Z_{k+1} - \Pi(x_{k+1}))$ // dual update
- update ρ
- k=k+1

Until convergence

(note: add 2 lines of code to Bundle Adjustment)

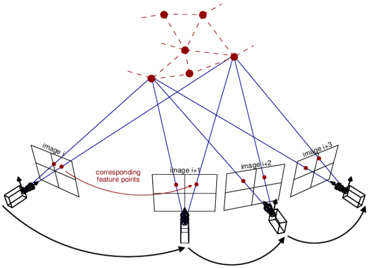
Pseudo-Convex Proximal Splitting

- A Meta-Algorithm
- Simple to implement
- Seamlessly transition between different residual norms
- Converges to a global minima
- Efficient

Overview

3 - Resection-Intersection [14]

[14] A Fast Resection-Intersection Method for the Known Rotation Problem.
Q. Zhang, T.-J. Chin, H. Le. CVPR 2018.



- A fast algorithm for solving the L_∞ Known Rotation Problem.
- A first order method (Gradient descent)
- Parallelizable

Known Rotation Problem

Solve:

$$\min_{\{t,x\}\in\mathcal{C}} \quad \max_{j,k} \left\| u_{j,k} - \frac{R_j^{1:2}x_k + t_j^{1:2}}{R_j^3 x_k + t_j^3} \right\|_p \qquad \text{(KRot)}$$

with $\mathcal{C} = \left\{ x, t \,|R_j^3 x_k + t_j^3 > 0, \;\; \forall j, k \right\}$

Observation:

- For fixed x_k (KRot) is separable, i.e. we can solve for t_j (Resectioning) separately for each camera

$$\min_{t_j} \quad \max_{k} \left\| u_{j,k} - \frac{R_j^{1:2}x_k + t_j^{1:2}}{R_j^3 x_k + t_j^3} \right\|_p \qquad \text{(Resectioning)}$$

- For fixed t_j (KRot) is separable, i.e. we can solve for x_k (Intersection) separately for each 3D point,

$$\min_{x_k} \quad \max_{j} \left\| u_{j,k} - \frac{R_j^{1:2}x_k + t_j^{1:2}}{R_j^3 x_k + t_j^3} \right\|_p \qquad \text{(Intersection)}$$

Resection-Intersection - Subproblems

Known-Rotation problem

$$\min_{\{t,x\}\in\mathcal{C}} \quad \max_{j,k} \left\| u_{j,k} - \frac{R_j^{1:2}x_k + t_j^{1:2}}{R_j^3 x_k + t_j^3} \right\|_p \tag{KRot}$$

Resulting (separable) subproblems on the form:

$$\begin{aligned} \min_{y\in\mathbb{R}^3} \quad & \max_i \ r_i(y) \\ \text{s.t.} \quad & c_i^T y + d_i > 0 \ \ \forall i, \end{aligned} \tag{sub}$$

$$r_i(y) = \frac{\|A_i y + b_i\|_p}{c_i^T y + d_i}$$

note: subproblems in $\mathbb{R}^3$

Solving Subproblems

$$\begin{aligned} \min_{y \in \mathbb{R}^3} \quad & \max_i \frac{\|A_i y + b_i\|_p}{c_i^T y + d_i} \\ \text{s.t.} \quad & c_i^T y + d_i > 0 \;\; \forall i, \end{aligned} \tag{sub}$$

[SolveSub]

Input: $\{A_i, b_i, c_i, d_i\}_{i=1}^N$, initial solution $\hat{y}$.

Repeat

- $\lambda \leftarrow$ Find descent direction using data and $\hat{y}$ (minimum closing ball (MEB))
- $\alpha \leftarrow$ Find step size using data, $\hat{y}$ and λ
- Update estimate: $\hat{y} \leftarrow \hat{y} + \alpha\lambda$

Until convergence

Resection-Intersection

[Main Algorithm]

Input: $\{R_j\}_{j=1}^{L}$, $\{u_{j,k}\}_{j=1,k=1}^{L,\ M}$.

Initialise: $\{t_j\}_{j=1}^{L}$ and $\{x_k\}_{k=1}^{M}$.

Repeat

- For each $k = 1, \ldots, M$, update x_k via [SolveSub].
- For each $j = 1, \ldots, L$, update t_j via [SolveSub].

Until convergence

Resection-Intersection - Summary

- First-order method (Block Gradient Descent)
- Problem is separable in t and x
- Subproblems small $\mathbb{R}^3$
- Efficient methods for finding descent directions and step-sizes
- Very fast

Algorithms - Summary

- Interior-Point methods
- Pseudo-Convex Splitting
- Resection-Intersection

Algorithms - Convergence Rate

1. Interior-Point methods
2. Pseudo-Convex Splitting
3. Resection-Intersection

Algorithms - Simplicity

1. Pseudo-Convex Splitting
2. Resection-Intersection
3. Interior-Point methods

Algorithms - Speed

1. Resection-Intersection
2. Interior-Point methods
3. Pseudo-Convex Splitting

Conclusions

- L_∞ and the Structure from Motion Pipeline
- Rotation Averaging & the Known Rotation Problem
- Pseudo-Convexity of L_∞ formulations
- Three methods for large-scale L_∞ optimization

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