Local Search and the Traveling Salesman Problem: A Feature-Based Characterization of Problem Hardness

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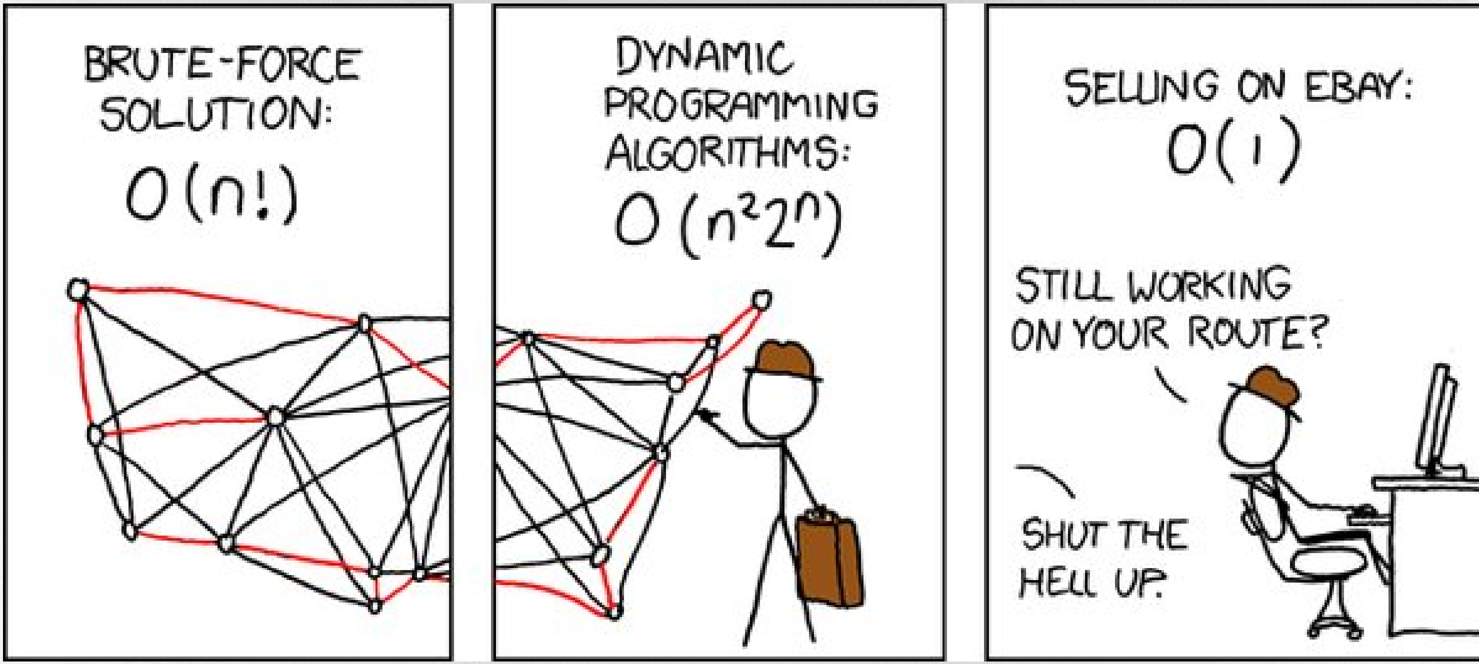
The Traveling Salesman Problem (TSP)



Aim: Predict Hardness of TSP instances

Problem Hardness: Two options

Number of swaps/iterations/...

Used in Smith-Miles et al. (2010)

Approximation quality

$$= \frac{\text{Expected solution tour length}}{\text{Optimal tour length}}$$

Characterize TSP instances

Requirement

All features can be computed without knowledge of the optimal tour.
Eliminates some (interesting) features.

Challenges

Normalization, dependence on # of nodes / edges

Characterize TSP instances

Taken from literature

Literature used

Smith-Miles et al. (2010), Kanda et al. (2011) and Smith-Miles and van Hemert (2011)

Classes of features

- Nearest Neighbor Distance (NNDs)
- Clustering
- Edge Costs / Distance Matrix

Focus on 2-opt (Croes, 1958) algorithm.

Reasons

- ▷ Historically first successful local search method for TSP
- ▷ Easy to understand
- ▷ Some progress on theoretical analysis
  (Chandra et al., 1999 and Englert et al., 2007)

Where do the TSP instances come from?

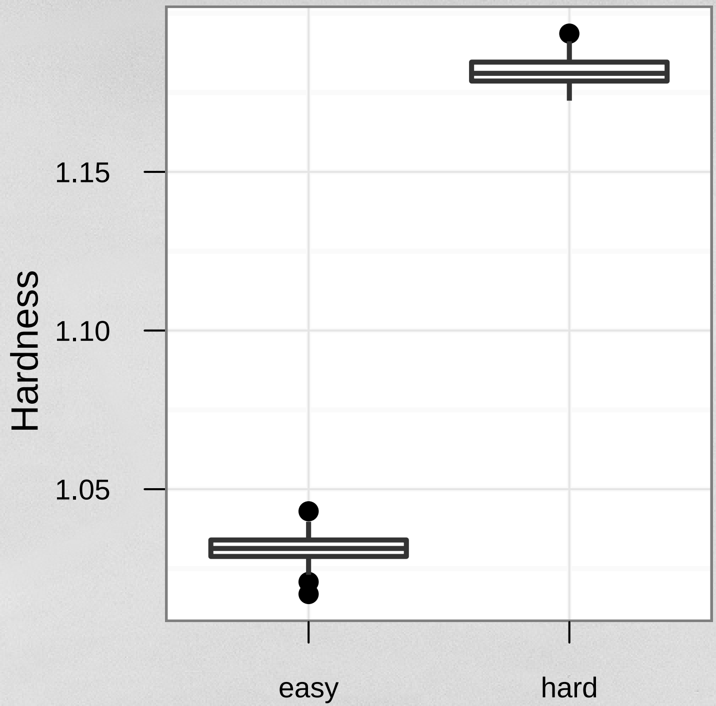
Instance Generator: EA

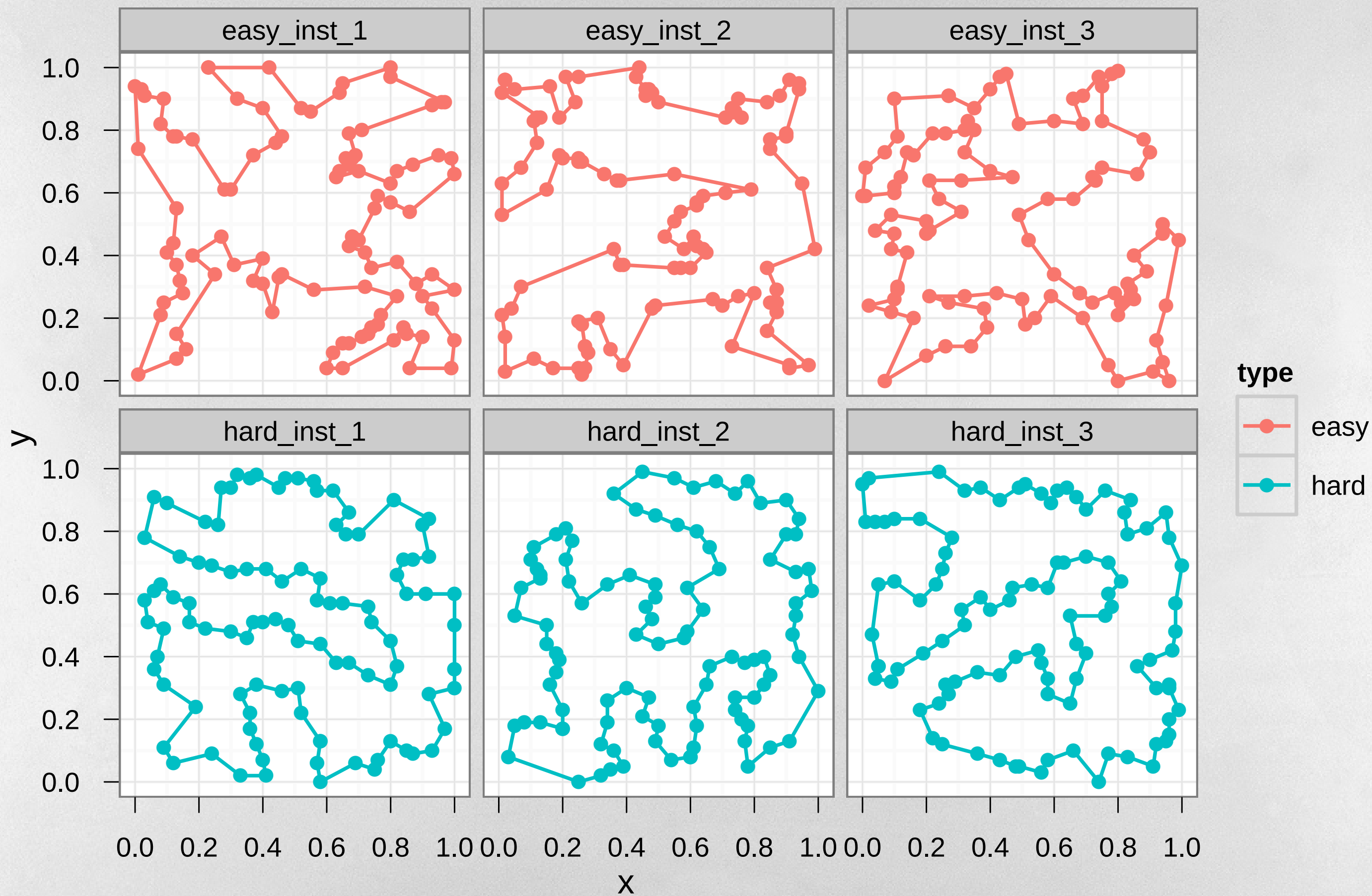
```
function tsp_generator(popSize=30, instSize=100, poolSize=50,
                       digits=2, repetitions=500):
  pop = randomInstances(popSize, instSize)
  while not done:
    fitness = computeFitness(pop, repetitions)
    matingPool = tournamentSelection(pop, poolSize, fitness)
    nextPop[1] = pop[whichBest(fitness)]
    for k = 2 to popSize:
      parent1, parent2 = randomElements(2, matingPool)
      offspring = uniformCrossover(parent1, parent2)
      nextPop[k] = round(
        uniformMutation(normalMutation(offspring)), digits)
    pop = nextPop
```

Use EA to generate 100 easy and hard instances

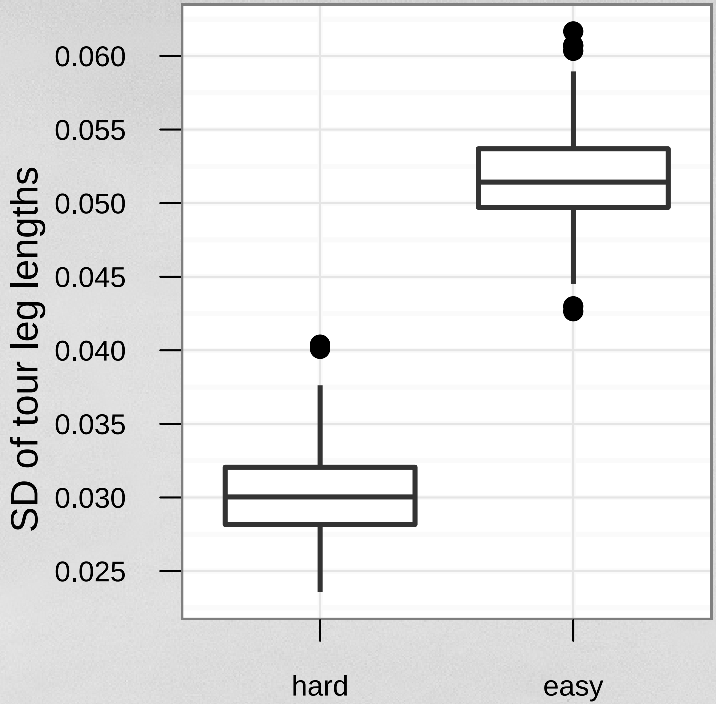
Problems

- ▷ Fitness function expensive
- ▷ Lots of manual tuning of EA
- ▷ Some runs hung


Hardness
1.15
1.10
1.05
easy
hard


easy_inst_1
easy_inst_2
easy_inst_3
hard_inst_1
hard_inst_2
hard_inst_3
1.0
0.8
0.6
0.4
0.2
0.0
y
x
type
easy
hard

Observation



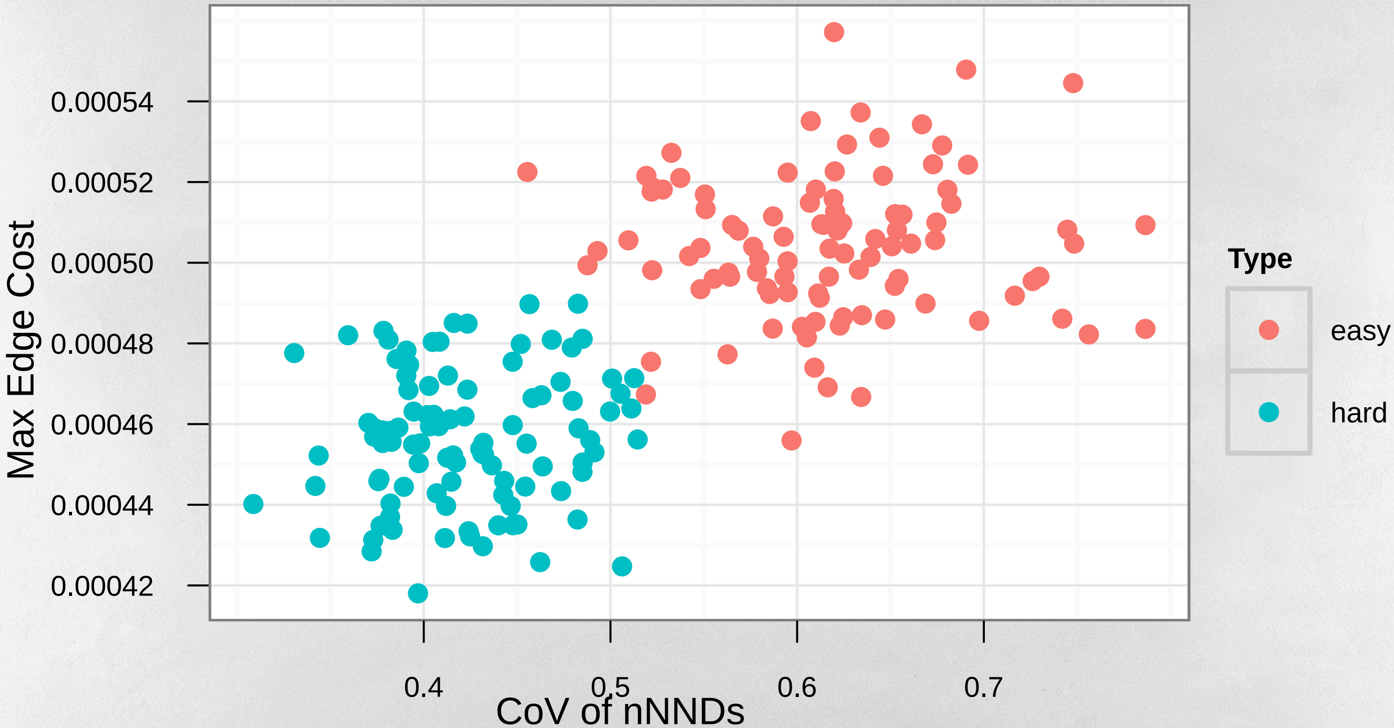
1 Tour leg lengths differ less for hard instances.

Prediction

- ▷ Calculate all features for the 200 instances
- ▷ Use decision tree (CART) to predict instance type

```
coefficient_of_variation_of_nnds >= 0.5167739 → easy
coefficient_of_variation_of_nnds < 0.5167739
   highest_edge_cost >= 0.000485 → easy
   highest_edge_cost < 0.000485 → hard
```

10-fold CV error rate: 3.02%



Max Edge Cost
0.00054
0.00052
0.00050
0.00048
0.00046
0.00044
0.00042
0.4
0.5
0.6
0.7
CoV of nNNDs
Type
easy
hard

This was an ``easy” task.

Instances chosen to be maximally different!

Morphing instances

We are missing instances that are between the two classes.

Idea

Create convex combination of an easy I_e and a hard instance I_h

$$I_n = \alpha I_e + (1 - \alpha) I_h \quad \text{with} \quad \alpha \in [0,1]$$

Morphing instances

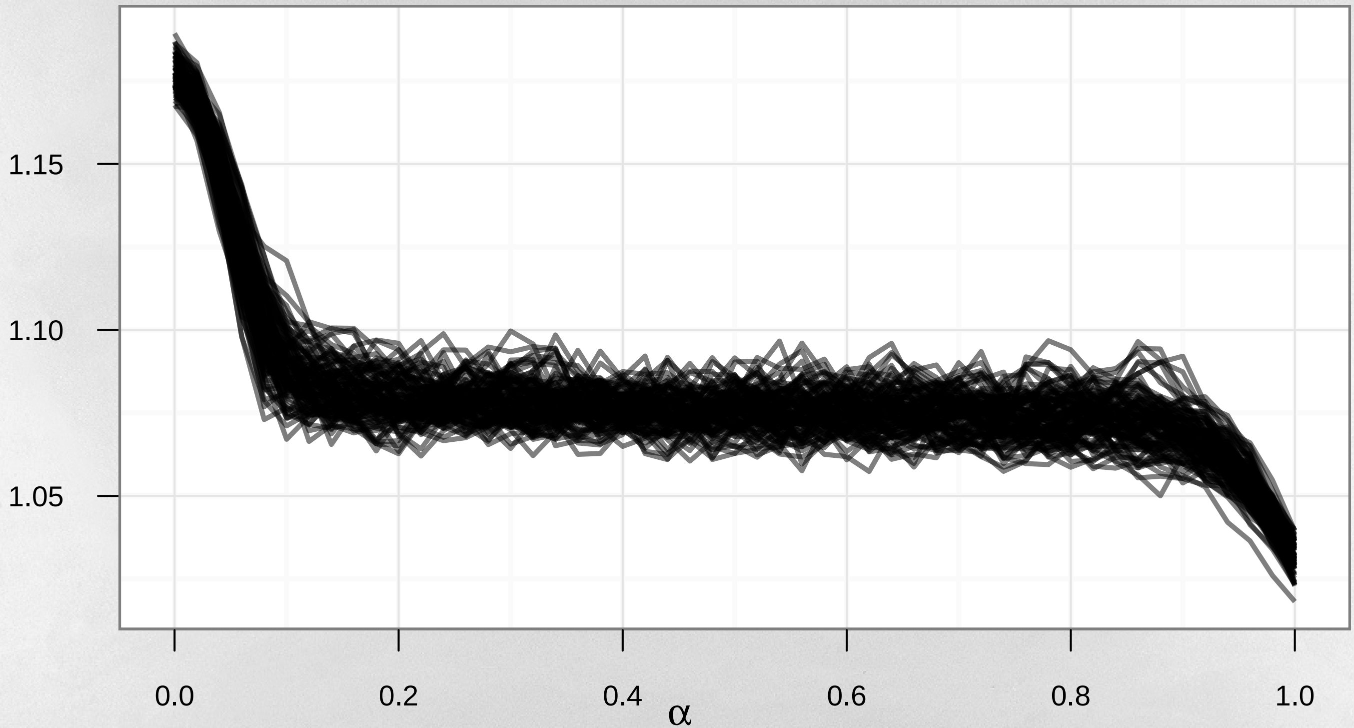
Possible Improvements

Match up points to minimize movement

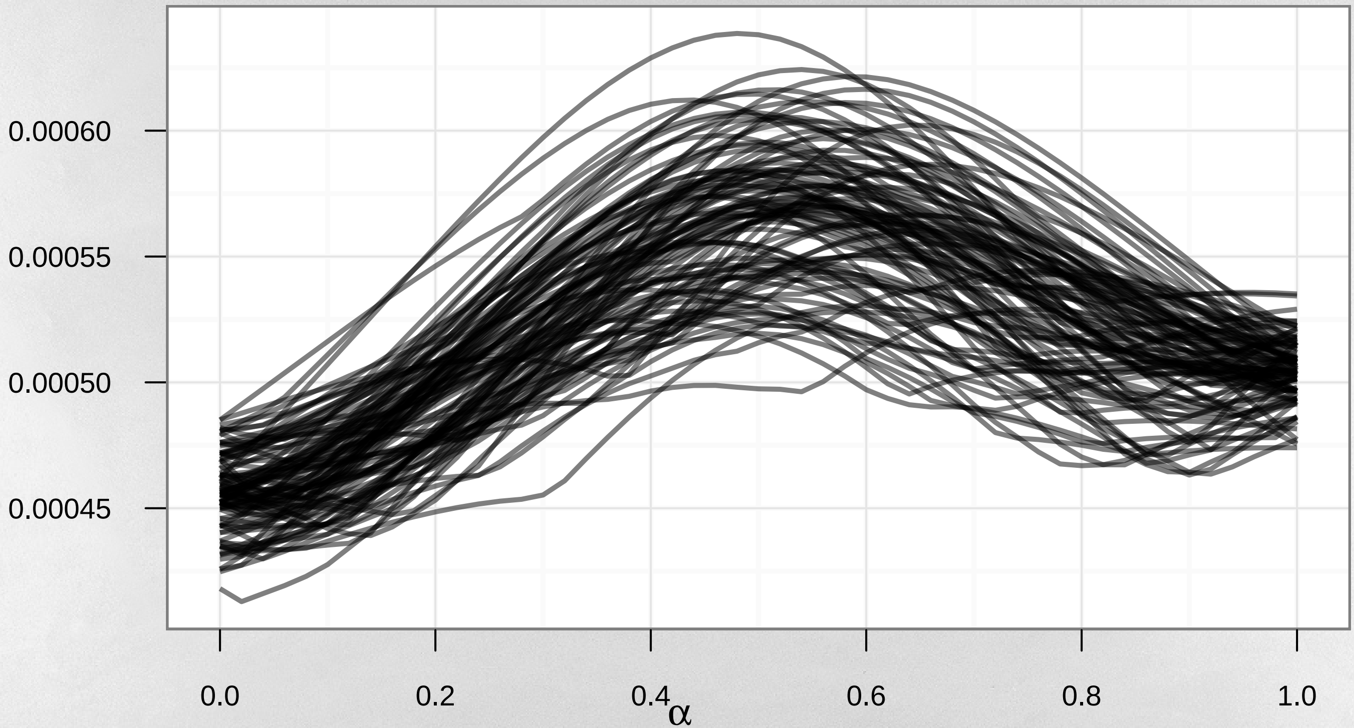
Usage

- ▷ For every combination of instances generate morph
- ▷ Calculate features for different α $(0.2, 0.4, ..., 0.8)$

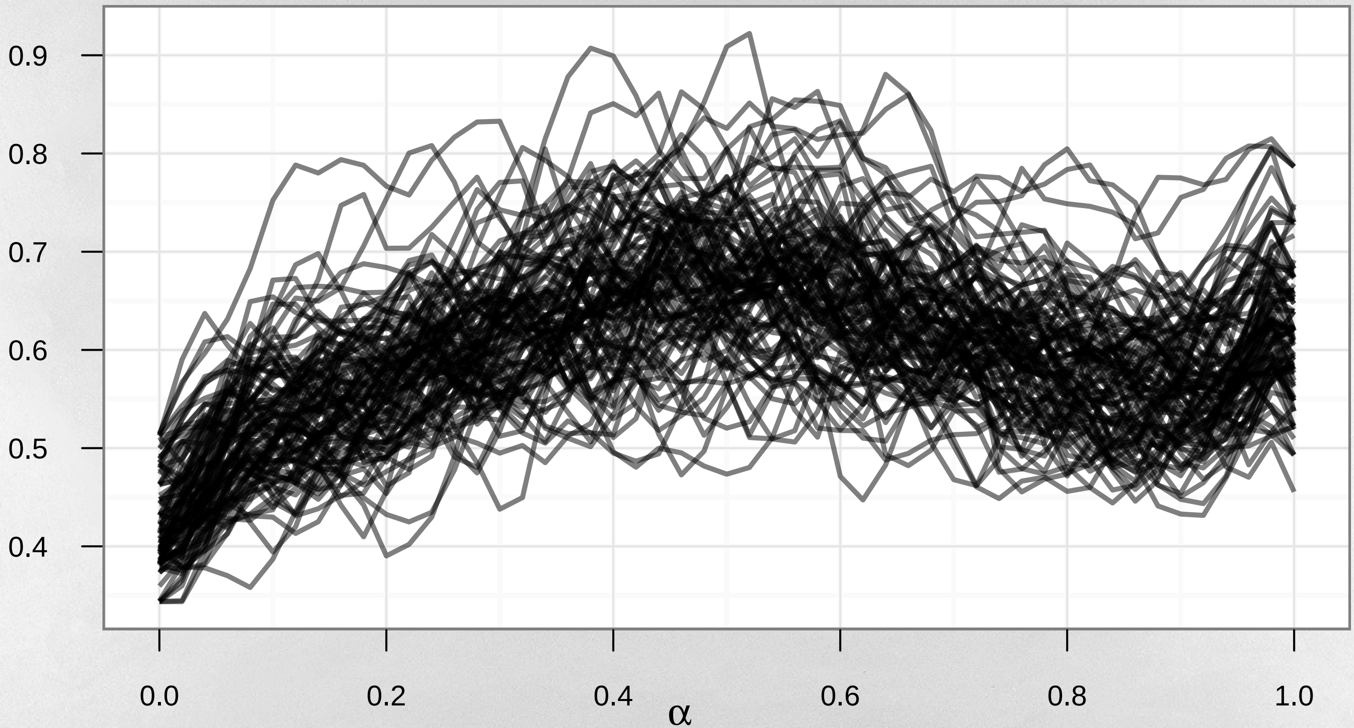
Problem Hardness



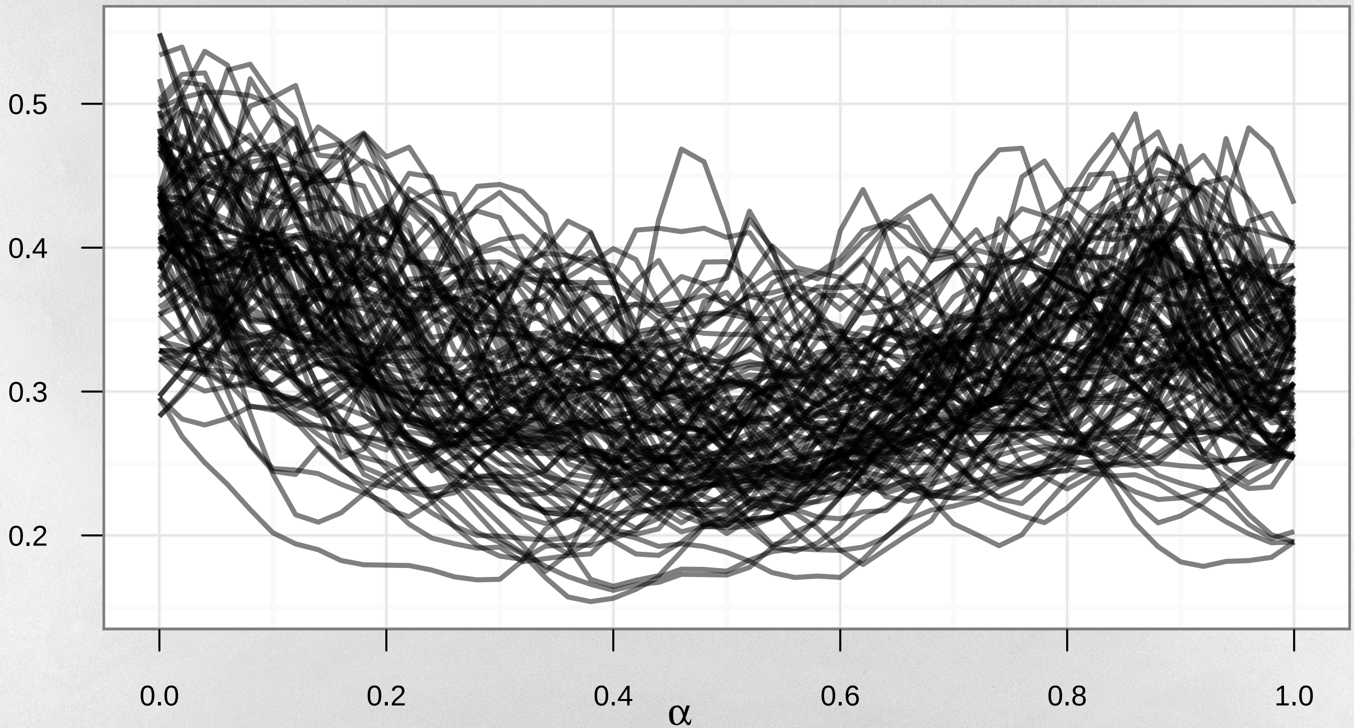
Max Edge Cost



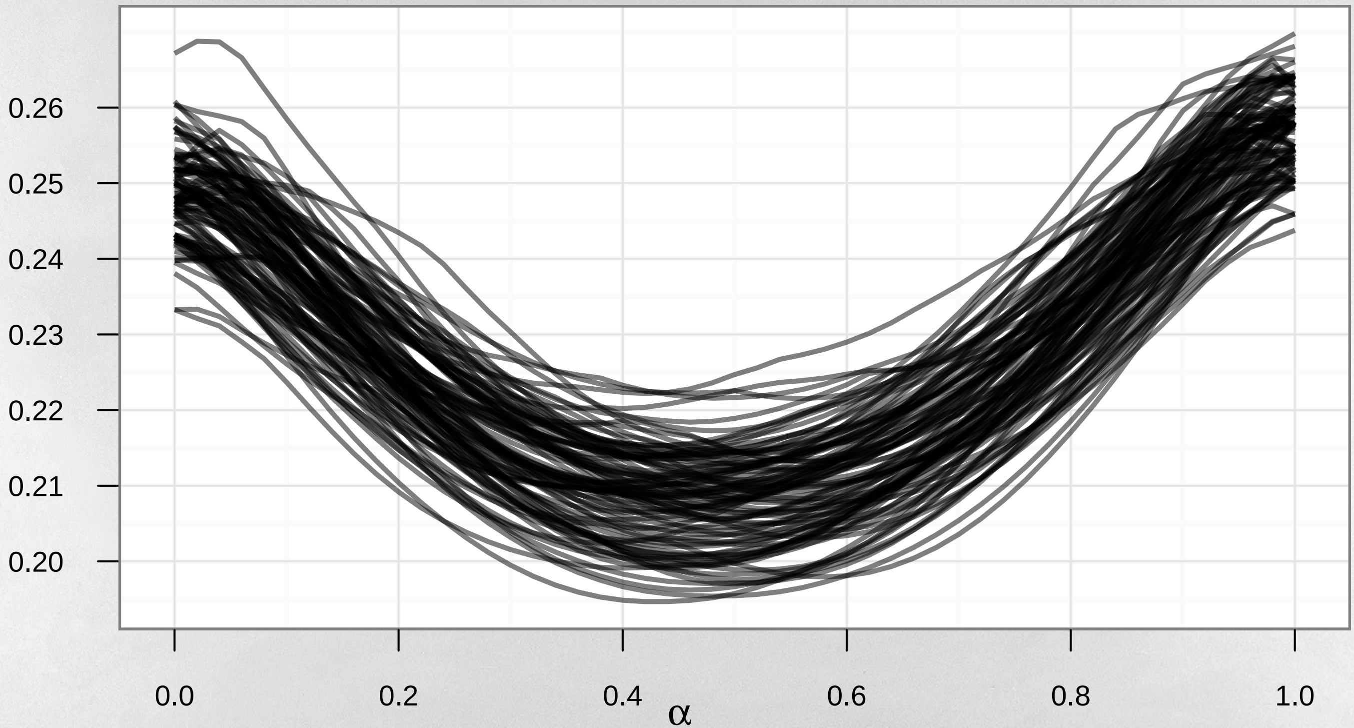
CoV of nNNDs



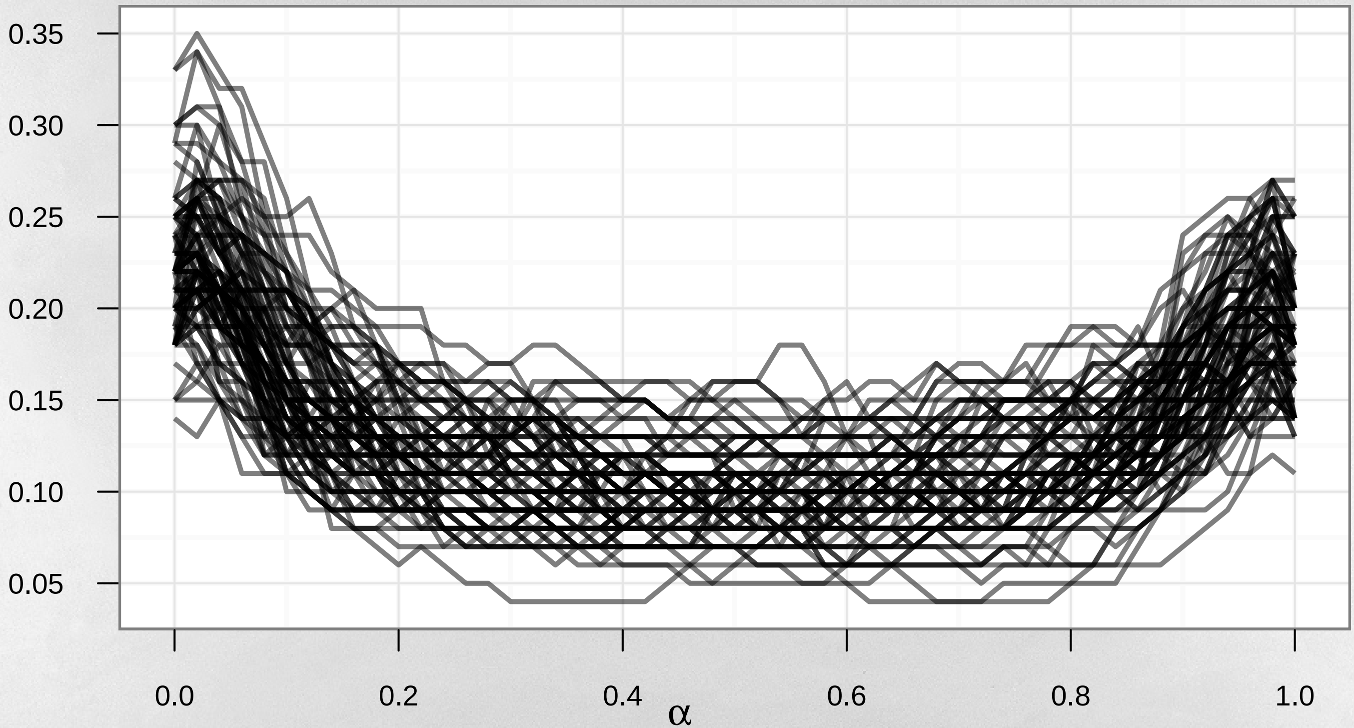
Mean of nNNDs



Variation of Edge Cost



Ratio of Cities near Edge



Prediction

Fit MARS model to data.

- ▷ Only use subset of morph results
- ▷ Do SFS to select subset of variables

RMSE estimated via 3-fold CV: 0.0113

Interpretation

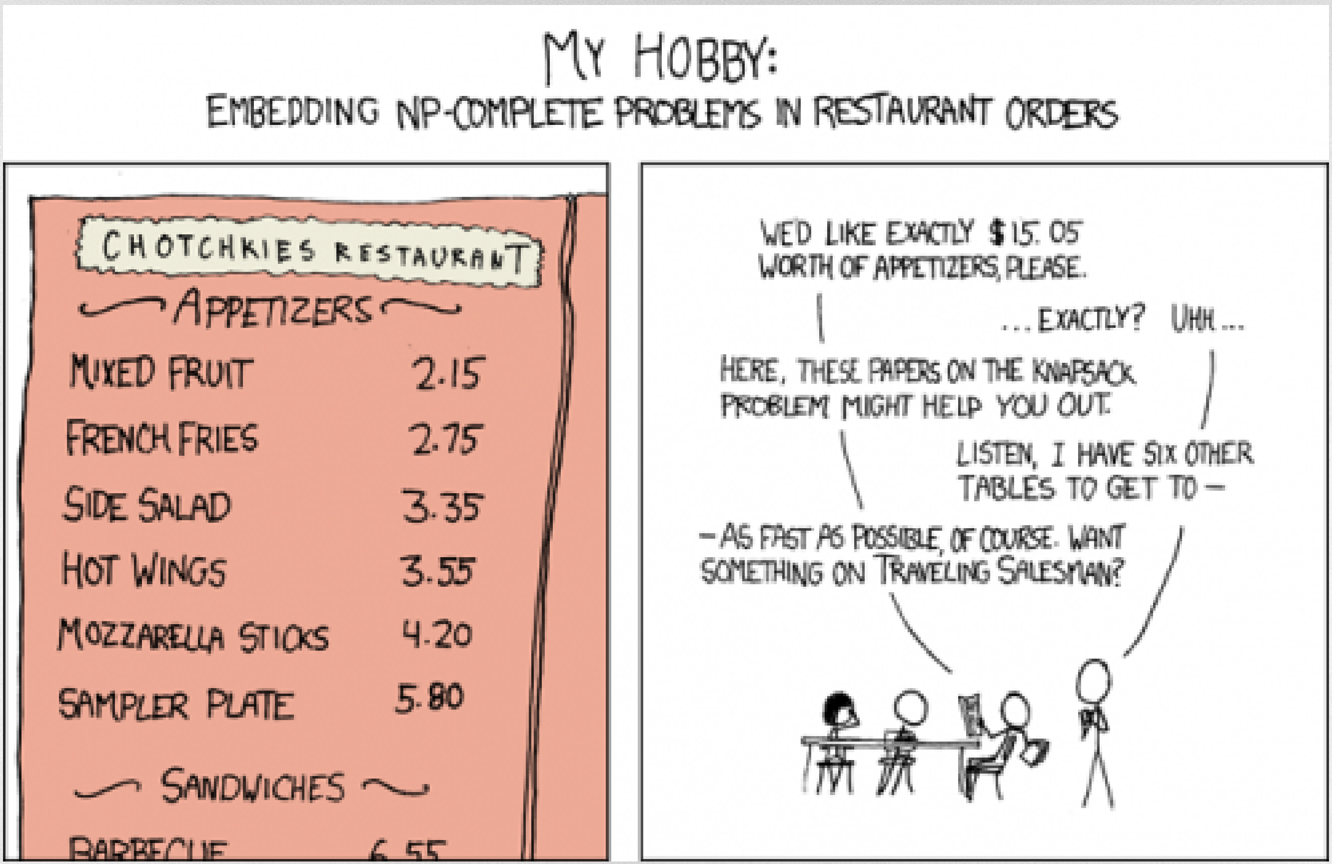
Not a black-box model. Please see paper for plots and interpretation.

Conclusion

- ▷ Generated ``easy” and ``hard” instances for 2-opt heuristic
- ▷ Characterized the instance sets using easily calculated features
- ▷ Showed novel approach to generate ``medium” instances (morphing)
- ▷ Predicted hardness of instance based on features using simple models

Outlook

- ▷ Optimize instance generation
- ▷ Study relation between features and theoretical properties of 2-opt
- ▷ Improve morphing
- ▷ Generate more diverse instance sets


MY HOBBY:
EMBEDDING NP-COMPLETE PROBLEMS IN RESTAURANT ORDERS
CHOTCHKIES RESTAURANT
APPETIZERS
MIXED FRUIT 2.15
FRENCH FRIES 2.75
SIDE SALAD 3.35
HOT WINGS 3.55
MOZZARELLA STICKS 4.20
SAMPLER PLATE 5.80
SANDWICHES
WED LIKE EXACTLY $15.05 WORTH OF APPETIZERS, PLEASE.
...EXACTLY? UHH...
HERE, THESE PAPERS ON THE KNAPSACK PROBLEM MIGHT HELP YOU OUT.
LISTEN, I HAVE SIX OTHER TABLES TO GET TO –
– AS FAST AS POSSIBLE, OF COURSE. WANT SOMETHING ON TRAVELING SALESMAN?