Relating Blob Model Complexity to Model Error

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Abstract

Many 3D Magnetotelluric models have very large number of parameters. Such large parameter spaces are difficult to search using non-deterministic search methods such as evolutionary algorithms. In recent work, we presented a description of 3D models using diffuse ellipsoid functions (blobs) and showed that this reduced-parameter description can be used in the derivation of good models via a hybrid evolutionary search process. However, this earlier work did not attempt to correlate the number of blobs used in the model with the model error eventually achieved. Knowledge of such a correlation would inform the number of blobs to use for the model. We investigated this relationship using the COMMEMI 3d2 model as a target. Preliminary results suggest a rough inverse relationship between the number of blobs and the model error (RMS). This relationship seems to exhibit diminishing returns for higher numbers of blobs.

Keywords: Magnetotellurics, Parametric-Models, Blobs, RMS

Introduction

Most model descriptions for Magnetotelluric (MT) inversion partition the model domain into a grid or mesh. For 3D models, these model descriptions can consist of many thousands of parameters. While such a large number of parameters is beneficial in terms of revealing fine detail, it can make a model unwieldy in terms of analysis and search. From the point of view of analysis, a very large number of parameters limits our ability to manipulate a model to correlate model features with model-error (RMS). For search, large parameter spaces limit the applicability of stochastic search methods such as evolutionary search.

Given these costs, and the inherent uncertainty attached to actual parameter values produced by the MT inversion process, the benefits of having so many parameters are diluted. In any case, common techniques to prevent over-fitting through regularisation such as the use of the ρ parameter in the Occam method[dH90] work to constrain the model space. A more dramatic reduction in model complexity can be achieved by describing models using simple functions. In past work, this has been done in terms of plates[JH79] and curved three-dimensional layers [Sch99]. These approaches have the dual advantages of a much-reduced search space and a much more concrete representation of model structure – a structure that is easier to interpret and manipulate. However, these earlier approaches focused on the discovery of simple, discrete, layered structures rather than general models.

In recent work [BA12] we used a 3D model description composed of diffuse ellipsoidal functions called *blobs*. In that work we demonstrated an inversion process combining a custom greedy search algorithm (Covariance Matrix Adaptation-Evolutionary Strategies (CMA-ES)[HC03]) and blob-models that produced good approximations of artificial and real models. However we did not investigate the relationship between model complexity, in terms of the number of blobs, and model error in terms of RMS. The work presented here explores the tradeoffs between to inform a-priori tradeoffs between the number of blobs and model error.

Method

We compose models from a small number of, usually overlapping, diffuse 3D ellipsoid functions (blobs) embedded in a background half-space of a predeterimined level. Each blob is described by 11 real-valued parameters defining, respectively, the central resistivity, attenuation (fuzziness), position (3 parameters), size (3 parameters) and orientation (3 parameters). These parameter settings are illustrated in Fig 1.

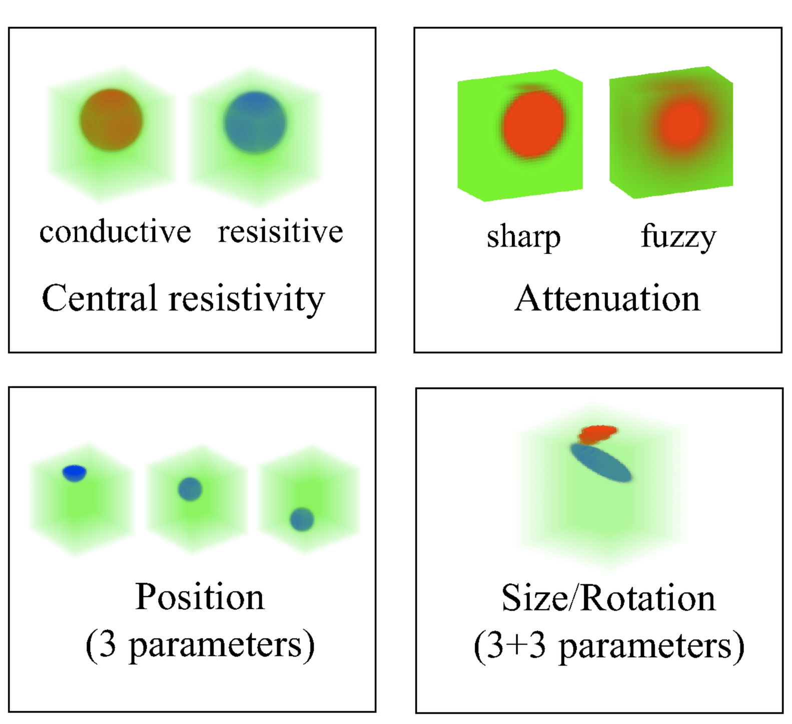

Figure 1: Illustration of blob parameters

We create models by sampling our ellipsoid functions into a hexahedral mesh representing the model space. Samples are measured by a deviation (positive or negative) from the half-space resistivity. Where blobs overlap we combine sampled deviations using a Hölder mean with a high negative power (e.g. -11). This high power weights the value at any sampled location heavily in favour of the blob with the most extreme deviation from the half-space at that location. The effect of this can be seen in Fig 2 where the conductive wedge (red) has a higher deviation from the half-space than the resistive area (blue) it intersects– allowing it to dominate the overlapping volume.

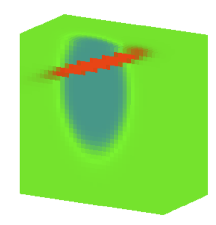

Figure 2: Overlap with a conductive blob dominating

In this work models are evaluated by running Siripunvaraporn's data-space forward model code[SELU05] on the mesh and field data.

Our MT Inversion process has four stages where candidate models are repeatedly evaluated and refined. Stage one, ***blob-seeding***, injects spherical blobs of different sizes and resistivities into the model one at a time. A custom greedy search optimises each blob for location and central intensity. Stage two, ***blob-priming***, uses the same greedy search process on rotating sets of the other parameters. After stage two it is possible to have some blobs worsen model error. The third stage, ***culling***, removes these blobs leaving only blobs that improve RMS. The fourth and final stage uses an evolutionary search method, CMA-ES, to fine-tune the model. This stage is usually the longest-running but has been highly beneficial in both reducing RMS and in improving model topology.

The experiments shown here measure the effect of different numbers of blobs on the RMS achieved by our search process. Our target model is the COMMEMI 3d2 model[MSM94], shown in cross-section in the first part of Fig 5. Response data for these experiments was derived by running forward modelling on the target model with 50 simulated stations in a rectangular grid. Data was collected for five frequencies ranging from 2Hz to 0.0001Hz. Error values of 5% were added for the diagonal components, and 50% for the off-diagonal components of the impedance tensors. The half-space was $10\Omega m$ (matching the shallow background of the target model). To speed the evolutionary process, we sampled into a small 13x14x17 model (where the z dimension of 17 includes seven air-layers). This low model resolution allowed forward modelling to run in just a few second on the 3.47GHz Intel Xeon processors used in our experiments.

We ran experiments for 1,2,3,4,6,8,12 and 16 blobs. Blobs were seeded into the model one at a time, alternating between resistive and conductive blobs, starting near the surface. The patterns of blob seeding are shown in Fig 3. Note how the deeper blobs are larger to account for the weaker field response they produce. The evolutionary process was run until the model RMS had converged or 100,000 evaluations had been done, whichever was soonest.

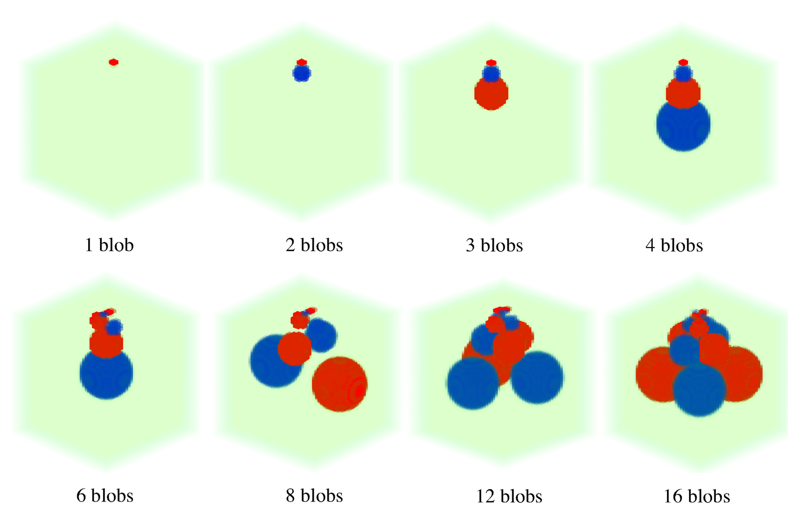

Figure 3: The starting (seeding) pattern of blobs in the model space.

Results

Fig 4 compares RMS error with the number of blobs for both the priming and CMA-ES stages.

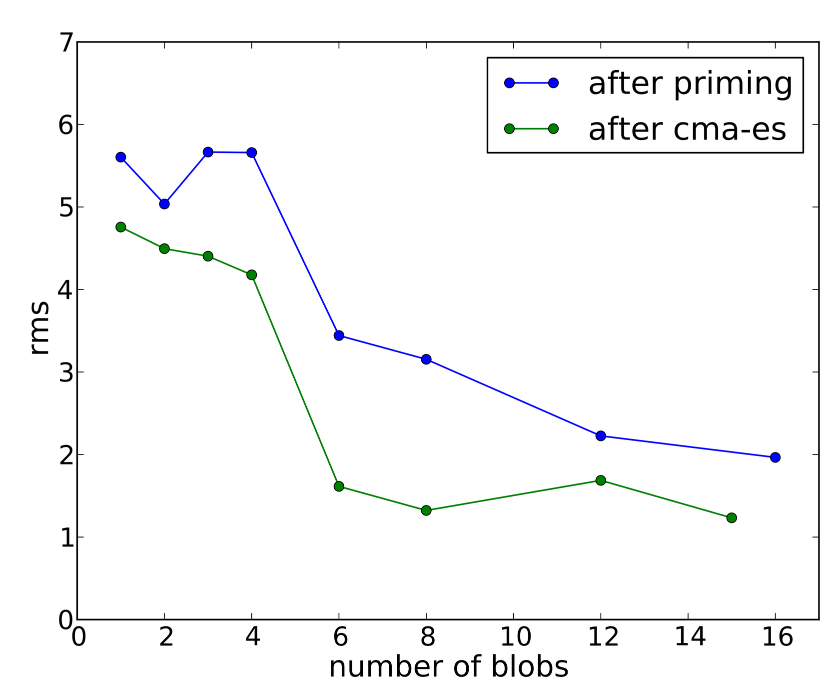

Figure 4: RMS vs the number of Blobs

Note that one blob was removed from the 16 blob model after the culling stage. Cross sections of models derived from these runs are shown in Fig 5.

Summary and Conclusion

The results in Fig 4 seem to indicate some ability to trade off model complexity against RMS as the number of blobs increases. Neither line is perfectly smooth, the roughness in the top line is in part due to some sensitivity of the current greedy search process to initial blob placement. The roughness of the bottom line could be least in part due to natural randomness in the CMA-ES search process. More runs are required to confirm the statistical significance of these results. If these results *are* sustained in later experiments, it may be that modellers can adjust the blob count to the point that RMS levels off in a similar manner to the adjustment of the ρ parameter on Occam models.

The derived models shown in Fig 5 indicate a very rough improvement in appearance as the number of blobs increases. The best approximation in terms of RMS and structure is the 16 blob model which was still slowly improving at the end of its run.

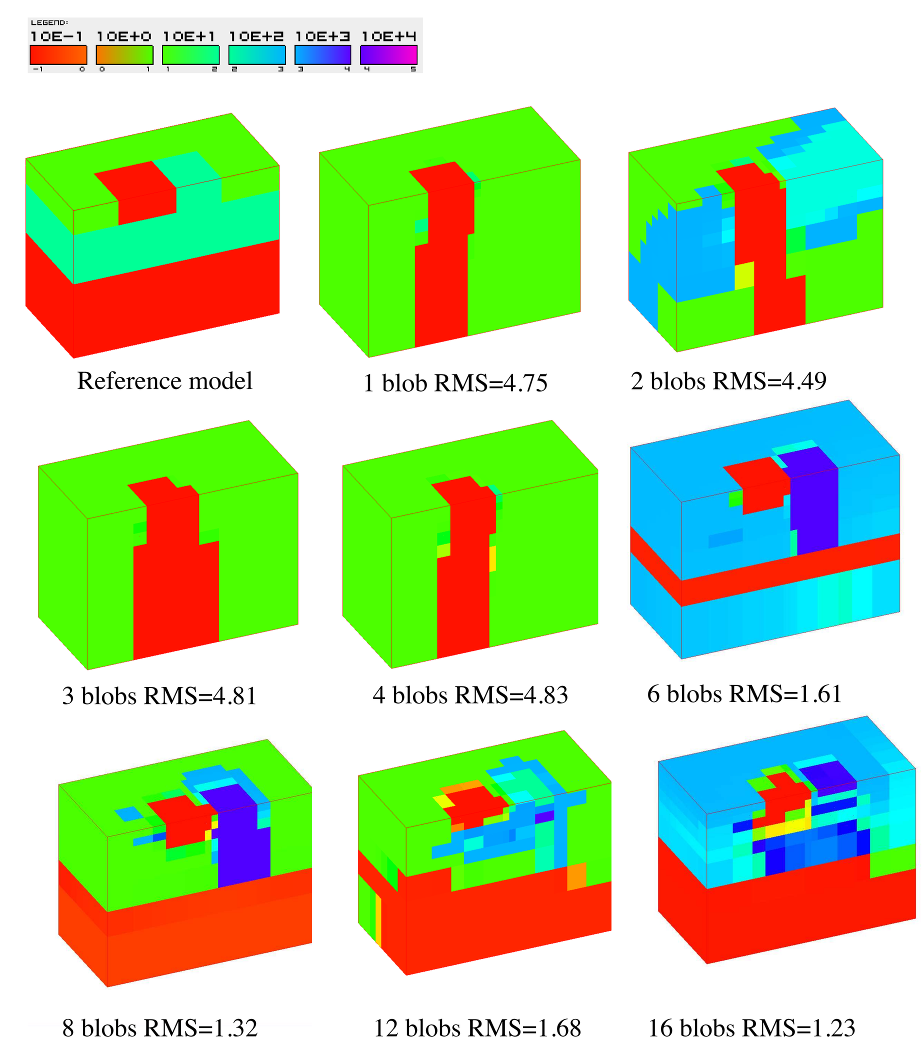

Figure 5: Model cross sections after CMA-ES

Our immediate future aim is to validate these results on multiple runs on different starting configurations. We would also like to manipulate the starting half-space parameter perhaps allowing it to evolve with the other parameters. We also intend to experiment with larger models and a finer-sampling interval, both of these measures will help smooth the search space and might make greedy search more effective.

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