Guaranteed Outlier Removal for Rotation Search

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Introduction

Recently, there is significant interest in developing globally optimal rotation search algorithms. A notable weakness of global algorithms, however, is their relatively high computational cost, especially on large problem sizes and data with a high proportion of outliers.

We present a **G**uarantee **O**utlier **R**emoval (**GORE**) algorithm suitable when rotations are computed on point matches. Capable to remove the majority of *wrong* point matches, GORE do not compromise optimality!

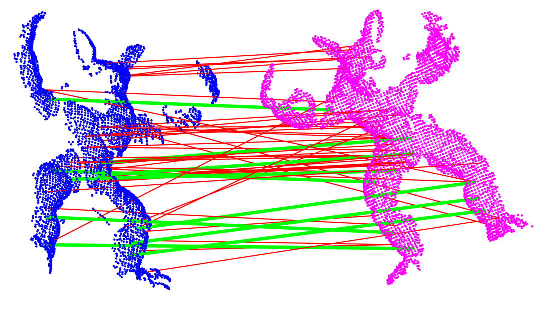
Used as a preprocessor to prune a large portion of the outliers from the input data, GORE enables substantial speed-up of rotation search algorithms.

GORE:

- Only removes **wrong** matches
- Is deterministic
- Accelerate optimal methods, and
- Is fast!

Source code available at `www.cs.adelaide.edu.au/~aparra`

Rotation Search



Given two point clouds related by a rotation, find the *best* rotation to align them.

Given a set of point matches $\mathcal{I} = \{(\mathbf{x}_i, \mathbf{y}_i)\}_{i=1}^{N}$, we aim solve for rotation inclusively when more than 90% of matches are incorrect. A robust methodology is *Consensus Set Maximisation*:

$$\begin{aligned} &\underset{\mathbf{R},\, \mathcal{I} \subseteq \mathcal{H}}{\text{maximize}} && |\mathcal{I}| \\ &\text{subject to} && \angle(\mathbf{R}\mathbf{x}_i, \mathbf{y}_i) \le \epsilon, \ \forall i \in \mathcal{I}, \end{aligned} \tag{1}$$

Outlier Removal

The rotation search problem (1) can be rewritten as

$$\underset{k \in \mathcal{H}}{\text{maximize}} \ f_k, \tag{19}$$

where f_k is defined as the maximum objective value of the subproblem P_k, with $k = 1, \ldots, N$:

$$\begin{aligned} &\underset{\mathbf{R}_k,\, \mathcal{I}_k \subseteq \mathcal{H} \setminus \{k\}}{\text{maximize}} && |\mathcal{I}_k| + 1 \\ &\text{subject to} && \angle(\mathbf{R}_k\mathbf{x}_i, \mathbf{y}_i) \le \epsilon, \ \forall i \in \mathcal{I}_k, \\ & && \angle(\mathbf{R}_k\mathbf{x}_k, \mathbf{y}_k) \le \epsilon. \end{aligned} \tag{P_k}$$

P_k seeks the rotation $\mathbf{R}_k$ that agrees with as many of the data as possible, given that $\mathbf{R}_k$ *must align* $(\mathbf{x}_k, \mathbf{y}_k)$.

Let $l \le |\mathcal{I}^*|$ be a lower bound for the solution of the rotation search problem (1). GORE depends on the ability to calculate an upper bound $\hat{f}_k$ for the result of each P_k. Given the lower and upper bound values, the following result can be established.

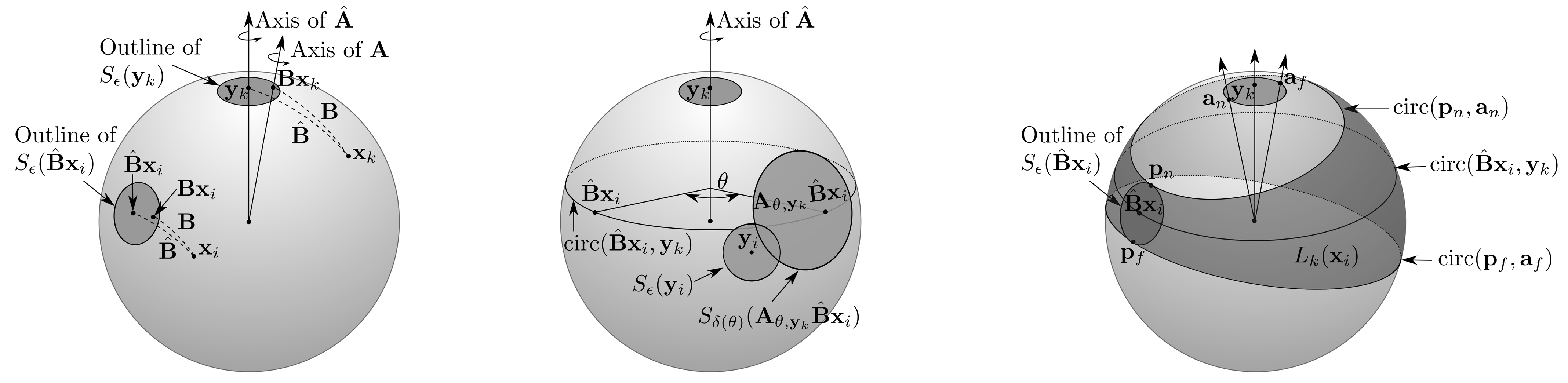
Theorem 1 *If $\hat{f}_k < l$, then $(\mathbf{x}_k, \mathbf{y}_k)$ is a true outlier, i.e., k does not exist in the solution $\mathcal{I}^*$ to* (1).

Algorithm

Require: Point matches $\{(\mathbf{x}_i, \mathbf{y}_i)\}_{i=1}^{N}$, inlier threshold ϵ.

1: $\mathcal{H} \leftarrow \{1, 2, \ldots, N\}$.
2: $\mathcal{H}' \leftarrow \mathcal{H}, \mathcal{O} \leftarrow \mathcal{H}, \mathcal{V} \leftarrow \emptyset$, and $l \leftarrow 0$.
3: **for all** $k \in \mathcal{O}$ **do**
4: $\quad \mathcal{V} \leftarrow \mathcal{V} \cup \{k\}$.
5: $\quad$ Compute upper bound $\hat{f}_k$ and suboptimal rotation $\tilde{\mathbf{R}}_k$ for problem P_k on data indexed by $\mathcal{H}'$.
6: $\quad \mathcal{C}_k \leftarrow \{i \mid i \in \mathcal{H}', \angle(\tilde{\mathbf{R}}_k\mathbf{x}_i, \mathbf{y}_i) \le \epsilon\}$.
7: $\quad l_k \leftarrow |\mathcal{C}_k|$.
8: $\quad$ **if** $l_k > l$ **then**
9: $\quad\quad l \leftarrow l_k$.
10: $\quad\quad \mathcal{O} \leftarrow \mathcal{H}' \setminus \mathcal{C}_k$.
11: $\quad$ **end if**
12: $\quad$ **if** $\hat{f}_k < l$ **then**
13: $\quad\quad \mathcal{H}' \leftarrow \mathcal{H}' \setminus \{k\}$.
14: $\quad$ **end if**
15: $\quad \mathcal{O} \leftarrow \mathcal{O} \setminus \mathcal{V}$.
16: **end for**
17: **return** $\{(\mathbf{x}_i, \mathbf{y}_i) \mid i \in \mathcal{H}'\}$.

Uncertainty Bound



$\mathbf{R}_k$ that solves P_k must bring $\mathbf{x}_k$ within angular distance ϵ from $\mathbf{y}_k$

$$\angle(\mathbf{R}_k\mathbf{x}_k, \mathbf{y}_k) \le \epsilon. \tag{2}$$

We interpret $\mathbf{R}_k$ by decomposing it into two rotations

$$\mathbf{R}_k = \mathbf{A}\mathbf{B} \tag{3}$$

where we define $\mathbf{B}$ as a rotation that honors the condition

$$\angle(\mathbf{B}\mathbf{x}_k, \mathbf{y}_k) \le \epsilon, \tag{4}$$

and $\mathbf{A}$ as a rotation about axis $\mathbf{B}\mathbf{x}_k$. Since $\mathbf{A}$ leaves $\mathbf{B}\mathbf{x}_k$ unchanged, (4) and hence (2) are always satisfied.

We aim to establish a bound on the position of $\mathbf{x}_i$ when acted upon by the set of feasible rotations $\mathbf{R}_k$. The set of $\mathbf{B}$ that maintain (4) cause $\mathbf{B}\mathbf{x}_k$ to lie within a spherical region of angular radius ϵ centered at $\mathbf{y}_k$

$$\mathbf{B}\mathbf{x}_k \in S_\epsilon(\mathbf{y}_k). \tag{5}$$

Since $\mathbf{B}\mathbf{x}_k$ is the rotation axis of $\mathbf{A}$, the interior of $S_\epsilon(\mathbf{y}_k)$ also represents the set of possible rotation axes for $\mathbf{A}$. Further, for any $i \ne k$

$$\angle(\mathbf{B}\mathbf{x}_i, \hat{\mathbf{B}}\mathbf{x}_i) = \angle(\mathbf{B}\mathbf{x}_k, \hat{\mathbf{B}}\mathbf{x}_k) = \angle(\mathbf{B}\mathbf{x}_k, \mathbf{y}_k) \le \epsilon. \tag{6}$$

Hence, the set of feasible $\mathbf{B}$ cause $\mathbf{B}\mathbf{x}_i$ to lie in a spherical region, i.e.,

$$\mathbf{B}\mathbf{x}_i \in S_\epsilon(\hat{\mathbf{B}}\mathbf{x}_i). \tag{7}$$

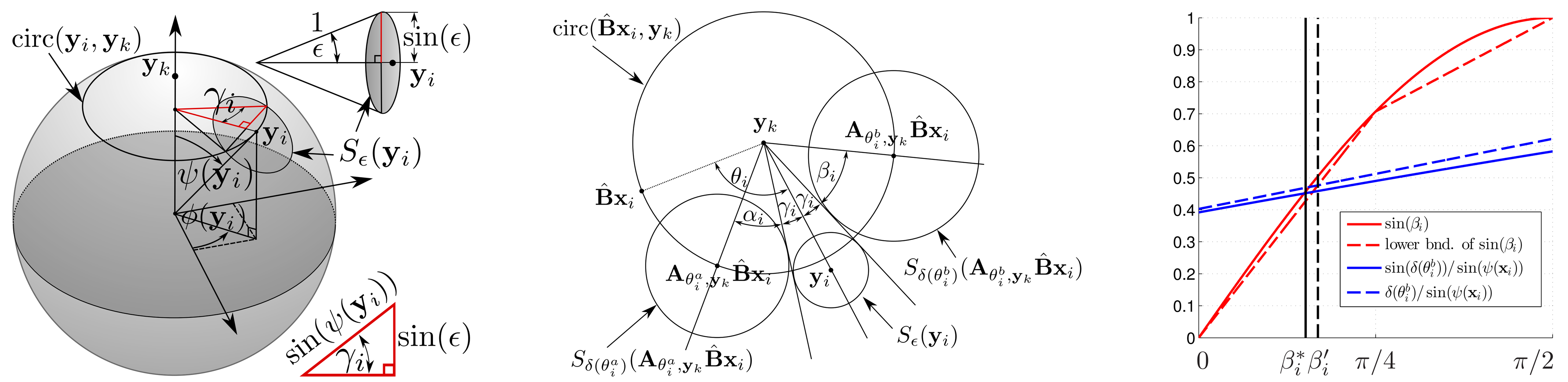
The bound on $\mathbf{R}_k\mathbf{x}_i$ can thus be analysed based on these two regions.

We denote $\mathbf{A}$ as $\mathbf{A}_{\theta,\mathbf{a}} := \exp(\theta\mathbf{a})$ and define $\text{circ}(\mathbf{p}, \mathbf{a}) := \{\mathbf{A}_{\theta,\mathbf{a}}\mathbf{p} \mid \theta \in [-\pi, \pi]\}$. The set of possible positions of $\mathbf{R}_k\mathbf{x}_i$ is then defined by

$$L_k(\mathbf{x}_i) := \{\text{circ}(\mathbf{p}, \mathbf{a}) \mid \mathbf{p} \in S_\epsilon(\hat{\mathbf{B}}\mathbf{x}_i), \mathbf{a} \in S_\epsilon(\mathbf{y}_k)\}. \tag{8}$$

Result 1 *For any $i \ne k$, if $S_\epsilon(\mathbf{y}_i)$ does not intersect with $L_k(\mathbf{x}_i)$, then $(\mathbf{x}_i, \mathbf{y}_i)$ cannot be aligned by any rotation $\mathbf{R}_k$ that satisfies* (2). *$(\mathbf{x}_i, \mathbf{y}_i)$ can then be safely removed without affecting the result f_k of P_k.*

Reducing the Uncertainty



For each $(\mathbf{x}_i, \mathbf{y}_i)$ that survives the pruning by Result 1, we reduce its uncertainty bound (8) into an *angular interval*. Consider rotating a point $\mathbf{p}$ with $\mathbf{A}_{\theta,\mathbf{u}}$ for a fixed angle θ and an axis $\mathbf{u} \in S_\epsilon(\mathbf{y}_k)$. We wish to bound $\mathbf{A}_{\theta,\mathbf{u}}\mathbf{p}$ given the uncertainty in $\mathbf{u}$

$$\max_{\mathbf{u} \in S_\epsilon(\mathbf{y}_k)} \angle(\mathbf{A}_{\theta,\mathbf{u}}\mathbf{p}, \mathbf{A}_{\theta,\mathbf{y}_k}\mathbf{p}) \le \max_{\mathbf{u} \in S_\epsilon(\mathbf{y}_k)} \|\theta\mathbf{u} - \theta\mathbf{y}_k\|_2 = 2|\theta|\sin(\epsilon/2). \tag{9}$$

Now we extend (9) to accommodate the uncertainty of $\mathbf{p} \in S_\epsilon(\hat{\mathbf{B}}\mathbf{x}_i)$

$$\begin{aligned} &\max_{\substack{\mathbf{p} \in S_\epsilon(\hat{\mathbf{B}}\mathbf{x}_i) \\ \mathbf{u} \in S_\epsilon(\mathbf{y}_k)}} \angle(\mathbf{A}_{\theta,\mathbf{u}}\mathbf{p}, \mathbf{A}_{\theta,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i) \\ \le &\max_{\substack{\mathbf{p} \in S_\epsilon(\hat{\mathbf{B}}\mathbf{x}_i) \\ \mathbf{u} \in S_\epsilon(\mathbf{y}_k)}} \angle(\mathbf{A}_{\theta,\mathbf{u}}\mathbf{p}, \mathbf{A}_{\theta,\mathbf{y}_k}\mathbf{p}) + \angle(\mathbf{A}_{\theta,\mathbf{y}_k}\mathbf{p}, \mathbf{A}_{\theta,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i) \\ \le &\ 2|\theta|\sin(\epsilon/2) + \epsilon. \end{aligned} \tag{10}$$

Define

$$\delta(\theta) = 2|\theta|\sin(\epsilon/2) + \epsilon. \tag{11}$$

(10) states that for a fixed θ and $\forall \mathbf{u} \in S_\epsilon(\mathbf{y}_k)$ and $\mathbf{B}\mathbf{x}_i \in S_\epsilon(\hat{\mathbf{B}}\mathbf{x}_i)$, the point $\mathbf{A}_{\theta,\mathbf{u}}\mathbf{B}\mathbf{x}_i$ lies in

$$S_{\delta(\theta)}(\mathbf{A}_{\theta,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i). \tag{12}$$

We wish to obtain a bound on the range of θ that enable $\mathbf{A}_{\theta,\mathbf{u}}\mathbf{B}\mathbf{x}_i$ to align with $\mathbf{y}_i$. This is analogous to seeking a bound on the θ that allows $S_{\delta(\theta)}(\mathbf{A}_{\theta,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i)$ to "touch" $S_\epsilon(\mathbf{y}_i)$.

Define $\phi(\mathbf{y}_i)$ and $\psi(\mathbf{y}_i)$ as the azimuth and inclination of $\mathbf{y}_i$. $S_\epsilon(\mathbf{y}_i)$ is contained between $\phi(\mathbf{y}_i) - \gamma_i$ and $\phi(\mathbf{y}_i) + \gamma_i$, where

$$\gamma_i = \arcsin\left(\frac{\sin(\epsilon)}{\sin(\psi(\mathbf{y}_i))}\right). \tag{13}$$

Let θ_i be the rotation angle such that $\mathbf{A}_{\theta_i,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i$ is on the meridian $\phi(\mathbf{y}_i)$.

Define $\Theta_i = [\theta_i^a, \theta_i^b]$, such that

$$\theta_i^a = \theta_i - \gamma_i - \alpha_i \quad \text{and} \quad \theta_i^b = \theta_i + \gamma_i + \beta_i, \tag{14}$$

where α_i is the largest value such that $S_{\delta(\theta_i^a)}(\mathbf{A}_{\theta_i^a,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i)$ still touches the meridian $(\phi(\mathbf{y}_i) - \gamma_i)$, and β_i is the largest value such that $S_{\delta(\theta_i^b)}(\mathbf{A}_{\theta_i^b,\mathbf{y}_k}\hat{\mathbf{B}}\mathbf{x}_i)$ still touches the meridian $(\phi(\mathbf{y}_i) + \gamma_i)$.

To determine Θ_i, we must find α_i and β_i. From (11),

$$\delta(\theta_i^a) = 2|\theta_i - \gamma_i - \alpha_i|\sin(\epsilon/2) + \epsilon \quad \text{and} \tag{15}$$

$$\delta(\theta_i^b) = 2|\theta_i + \gamma_i + \beta_i|\sin(\epsilon/2) + \epsilon. \tag{16}$$

From geometric considerations on the spherical regions,

$$\sin(\alpha_i) = \frac{\sin(\delta(\theta_i^a))}{\sin(\psi(\mathbf{x}_i))}, \quad \sin(\beta_i) = \frac{\sin(\delta(\theta_i^b))}{\sin(\psi(\mathbf{x}_i))}. \tag{17}$$

Solving (17) is non-trivial. However, since all that we require is a bounding interval Θ_i, we replaced the sine functions with that yield a valid bounding interval.

Interval stabbing Our upper bound $\hat{f}_k$ is obtained as the largest number of point matches that can be aligned by the same angle θ.

$$\hat{f}_k = 1 + \underset{\theta \in [-\pi,\pi]}{\text{maximize}} \sum_j \lfloor \theta \in [\theta_j^a, \theta_j^b] \rfloor. \tag{18}$$

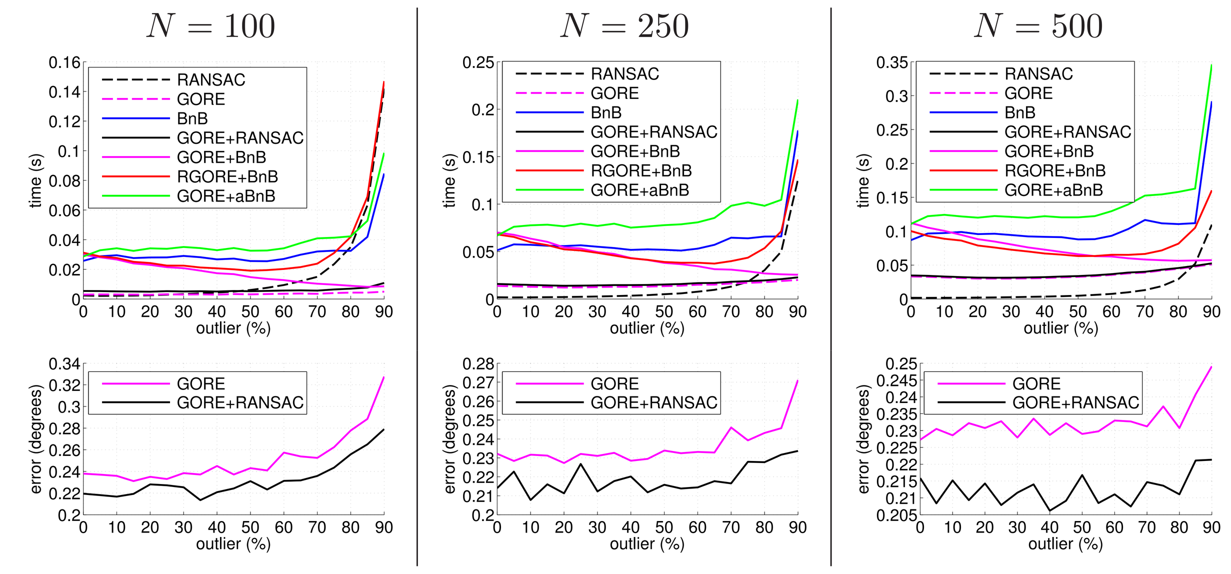
References

1. R. I. Hartley and F. Kahl, "Global Optimization through Rotation Space Search," International Journal of Computer Vision, vol. 82, no. 1.
2. L. Svarm, O. Enqvist, M. Oskarsson, and F. Kahl, "Accurate Localization and Pose Estimation for Large 3D Models," in Computer Vision and Pattern Recognition (CVPR), Columbus, 2014.

Conclusion

We have presented a guaranteed outlier removal technique for rotation search, in the sense that any datum it removes cannot be in the globally optimal solution. Based on simple geometric operations, our algorithm is deterministic and efficient. Experiments show that, by significantly reducing a significant amount of the outliers, our method greatly speeds up globally optimal rotation search.

Results on Synthetic Data



Results on Real Data

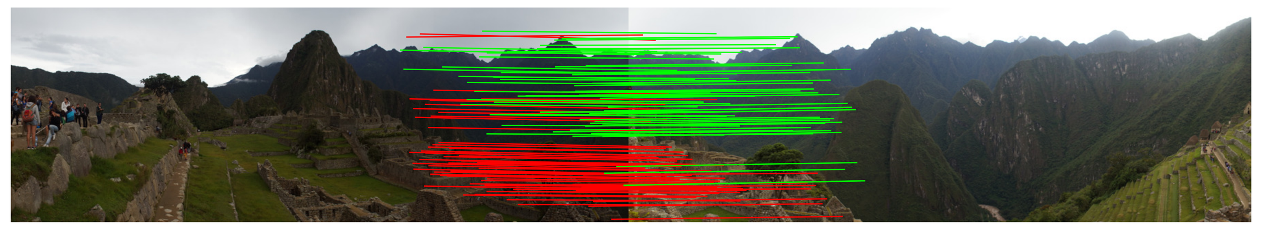
Results on point cloud registration

Object	N	irat	GORE lwbnd	GORE err (°)	GORE out	GORE time (s)	RANSAC lwbnd	RANSAC err (°)	RANSAC time (s)	BnB opt	BnB err (°)	BnB time (s)	RANSAC +BnB time (s)	GORE +BnB time (s)
buddha	100	0.09	6	0.23	53	0.009	7	0.36	0.164	9	0.37	0.225	0.389	0.074
$\lvert S_1\rvert = 4151$	250	0.05	9	0.31	178	0.040	10	0.24	0.583	12	0.22	0.980	1.561	0.116
$\lvert S_2\rvert = 3901$	500	0.03	13	0.35	390	0.112	14	0.31	1.366	17	0.27	2.875	4.211	0.237
	750	0.02	13	0.34	590	0.304	14	0.32	4.127	17	0.27	7.565	11.827	0.630
	1000	0.01	13	0.32	807	0.447	14	0.30	6.494	17	0.27	12.610	19.470	1.018
bunny	100	0.18	16	0.19	74	0.003	16	0.20	0.032	18	0.13	0.030	0.062	0.003
$\lvert S_1\rvert = 6533$	250	0.10	20	0.27	209	0.015	21	0.24	0.133	24	0.13	0.145	0.278	0.024
$\lvert S_2\rvert = 6226$	500	0.06	27	0.23	442	0.056	26	0.23	0.342	30	0.22	0.520	0.881	0.076
	750	0.04	31	0.18	684	0.127	29	0.25	0.659	32	0.23	1.245	1.946	0.147
	1000	0.04	32	0.19	924	0.219	30	0.24	1.220	35	0.14	2.445	3.764	0.269
armadillo	100	0.10	7	0.17	80	0.003	8	0.30	0.125	10	0.21	0.095	0.215	0.013
$\lvert S_1\rvert = 4508$	250	0.06	10	0.17	229	0.014	12	0.31	0.501	14	0.26	0.350	0.875	0.021
$\lvert S_2\rvert = 4362$	500	0.03	10	0.69	469	0.055	12	0.31	1.783	15	0.24	1.430	3.198	0.066
	750	0.02	13	0.34	713	0.146	13	0.29	3.270	16	0.24	3.435	7.002	0.161
	1000	0.01	13	0.34	958	0.233	13	0.31	6.843	16	0.50	7.150	14.505	0.264
dragon	100	0.20	19	0.22	71	0.004	18	0.20	0.024	20	0.24	0.060	0.079	0.014
$\lvert S_1\rvert = 5332$	250	0.12	29	0.11	205	0.016	29	0.15	0.068	30	0.25	0.175	0.241	0.034
$\lvert S_2\rvert = 4683$	500	0.07	30	0.18	446	0.055	31	0.17	0.257	33	0.22	0.565	0.827	0.065
	750	0.05	33	0.15	693	0.167	33	0.16	0.506	35	0.17	1.340	1.908	0.184
	1000	0.04	36	0.12	939	0.226	36	0.16	0.870	38	0.14	2.635	3.557	0.283

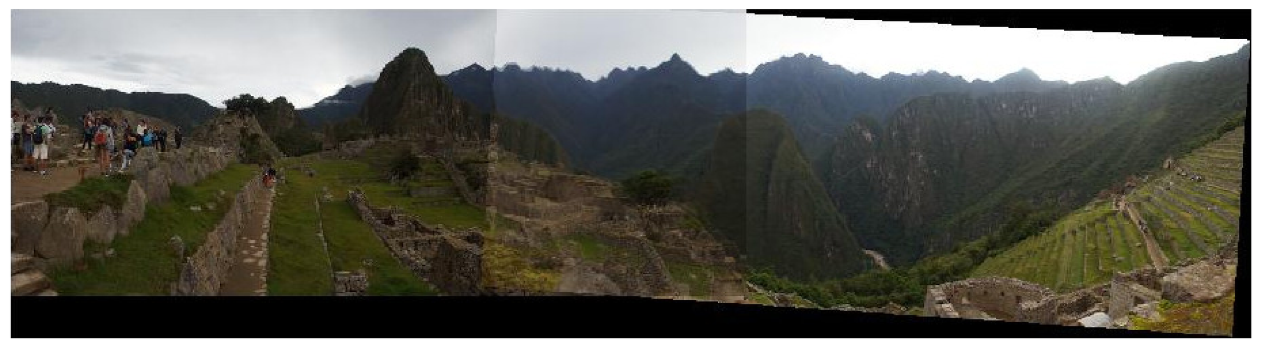
Results on image stitching

Image pair	N	irat	GORE lwbnd	GORE err (°)	GORE out	GORE time (s)	BnB opt	BnB time (s)	GORE +BnB time (s)
valparaiso	147	0.54	79	1.55	68	0.01	79	0.21	0.01
machupichu	194	0.33	51	1.10	60	0.02	64	1.94	1.27
paris1	718	0.07	49	0.01	584	0.33	51	16.68	2.03
paris2	921	0.22	181	0.16	311	1.07	200	16.24	11.33
rio	675	0.18	115	0.69	379	0.20	120	11.43	4.74

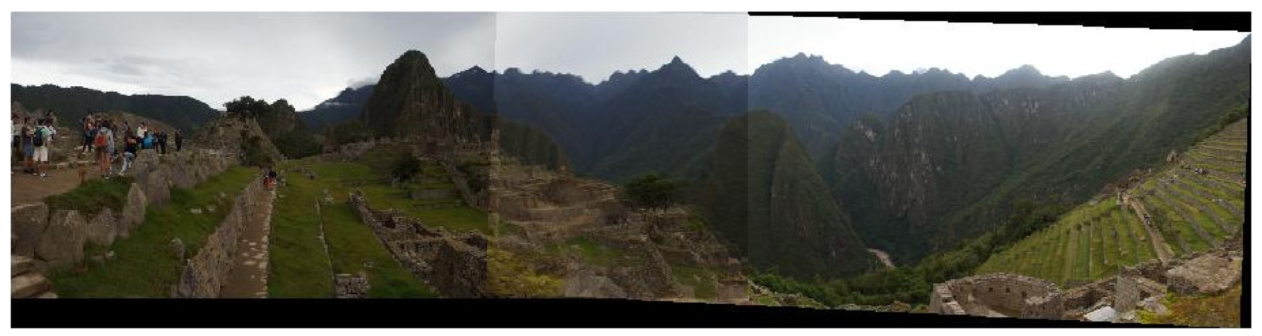
SIFT keypoint matches (green = true inliers, red = true outliers)



Matches that remain after preprocessing with GORE.



Stitching result using suboptimal rotation by GORE.



Stitching result using globally optimal rotation by BnB.