Fast Rotation Search with Stereographic Projections for 3D Registration

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Contribution

We construct a fast and global optimal rotation search algorithm with respect to an inherently robust geometric matching criterion. We propose a novel bounding function for branch and bound (BnB) that allows rapid evaluation. Underpinning our bounding function is the usage of stereographic projections to precompute and spatially index all possible point matches. This yields a robust and global algorithm that is significantly faster than previous methods.

Problem

Let $\mathcal{M} = \{\mathbf{m}_i\}_{i=1}^{M}$ and $\mathcal{B} = \{\mathbf{b}_j\}_{j=1}^{B}$ be two 3D point clouds related by a 3D rotation, find $\mathbf{R} \in SO(3)$ that maximises the geometric matching criterion

$$Q(\mathbf{R}) = \sum_i \max_j \lfloor \|\mathbf{R}\mathbf{m}_i - \mathbf{b}_j\| \le \epsilon \rfloor, \qquad (1)$$

where $\lfloor \cdot \rfloor$ is the indicator function.
This objective function is robust since two points are matched only if their distance is less than the inlier threshold ϵ.

Branch and Bound

The geometric matching criterion (1) is maximised by BnB over all rotations. A 3D rotation $\mathbf{R}$ is represented as a 3-vector $\mathbf{r} = \theta\hat{\mathbf{r}}$ where $\hat{\mathbf{r}}$ is the axis of rotation and θ the rotation angle. All rotations are thus contained in a π-ball.

BnB rotation search algorithm

Require: Point sets $\mathcal{M}$ and $\mathcal{B}$, threshold ϵ.

1: Initialise priority queue q, $\mathbb{B} \leftarrow$ cube of side 2π,
 $Q^* \leftarrow 0$, $\mathbf{R}^* \leftarrow null$.
2: Insert $\mathbb{B}$ into q.
3: **while** q is not empty **do**
4: Obtain a box $\mathbb{B}$ from q.
5: $\mathbf{R_c} \leftarrow$ centre rotation of $\mathbb{B}$.
6: If $Q(\mathbf{R_c}) = Q^*$ then terminate.
7: If $Q(\mathbf{R_c}) > Q^*$ then $Q^* \leftarrow Q(\mathbf{R_c})$, $\mathbf{R}^* \leftarrow \mathbf{R_c}$.
8: Subdivide $\mathbb{B}$ into $\mathbb{B}_l$ and $\mathbb{B}_r$.
9: If $\hat{Q}(\mathbb{B}_l) > Q^*$, insert $\mathbb{B}_l$ with priority $\hat{Q}(\mathbb{B}_l)$ into q.
10: If $\hat{Q}(\mathbb{B}_r) > Q^*$, insert $\mathbb{B}_r$ with priority $\hat{Q}(\mathbb{B}_r)$ into q.
11: **end while**
12: **return** Optimal rotation $\mathbf{R}^*$ with quality Q^*.

$\hat{Q}(\mathbb{B})$ is obtained from the known inequality

$$\angle(\mathbf{R_c}\mathbf{m}, \mathbf{R_u}\mathbf{m}) \le \max_{\mathbf{u}\in\mathbb{B}} \|\mathbf{c} - \mathbf{u}\| := \alpha_{\mathbb{B}}, \qquad (2)$$

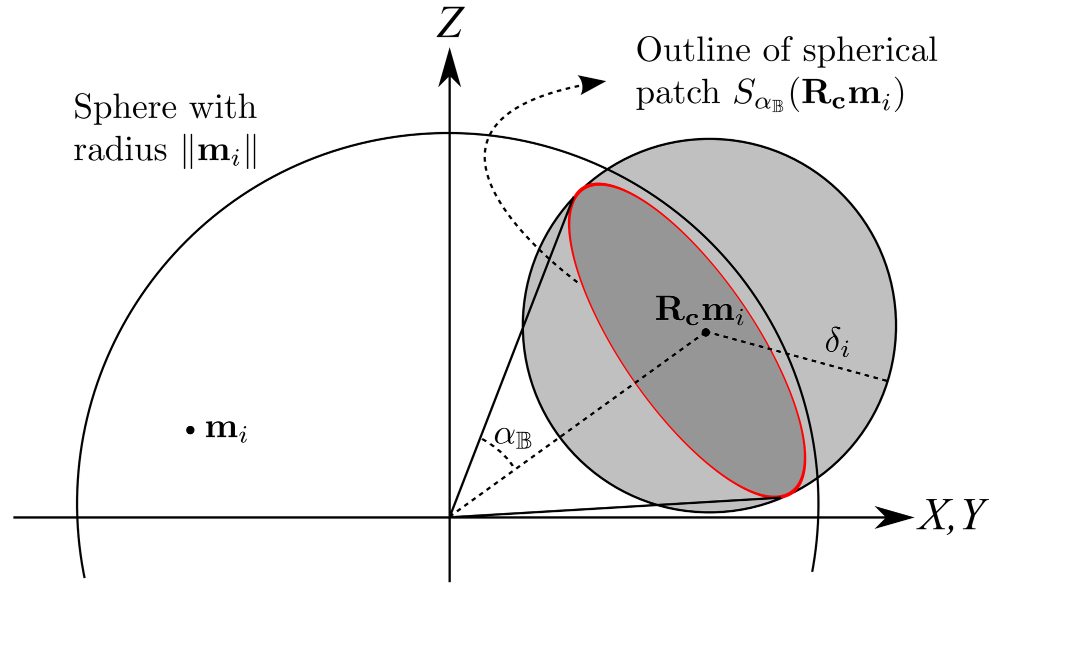
where $\mathbf{R_c}$ is the rotation at the centre of $\mathbb{B}$. Then, Breuel's upper bound is derived as

$$\hat{Q}_{br}(\mathbb{B}) = \sum_i \max_j \lfloor \|\mathbf{R_c}\mathbf{m}_i - \mathbf{b}_j\| \le \epsilon + \delta_i \rfloor, \qquad (3)$$

where $\delta_i = \|\mathbf{m}_i\|\sqrt{2(1-\cos(\alpha_{\mathbb{B}}))}$

Proposed Bound

Critical to convergence speed are the tightness and evaluation efficiency of $\hat{Q}(\mathbb{B})$.



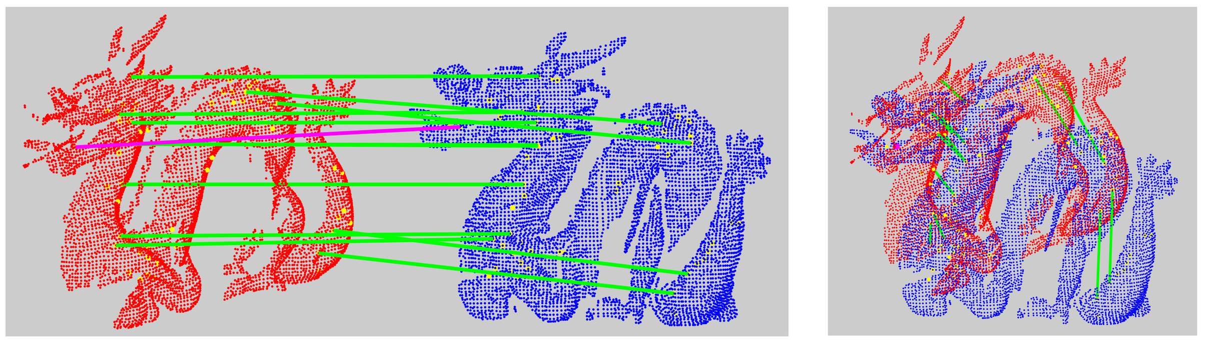
Let $l_\epsilon(\mathbf{b})$ the *solid* ball of radius ϵ centred at $\mathbf{b}$, then the geometric matching criterion can be rewritten as

$$Q(\mathbf{R}) = \sum_i \max_j \lfloor \mathbf{R}\mathbf{m}_i \in l_\epsilon(\mathbf{b}_j) \rfloor. \qquad (4)$$

Under all rotations in $\mathbb{B}$, $\mathbf{m}_i$ may lie only on the *spherical patch* $S_{\alpha_{\mathbb{B}}}(\mathbf{R_c}\mathbf{m}_i)$ centred at $\mathbf{R_c}\mathbf{m}_i$ with angular radius $\alpha_{\mathbb{B}}$, then the following bound is inferred

$$\hat{Q}_{sp}(\mathbb{B}) = \sum_i \max_j \lfloor S_{\alpha_{\mathbb{B}}}(\mathbf{R_c}\mathbf{m}_i) \cap l_\epsilon(\mathbf{b}_j) \neq \emptyset \rfloor. \qquad (5)$$

6DOF Framework



We conduct 6DOF registration experiments by using our rotation search algorithm in the following framework.

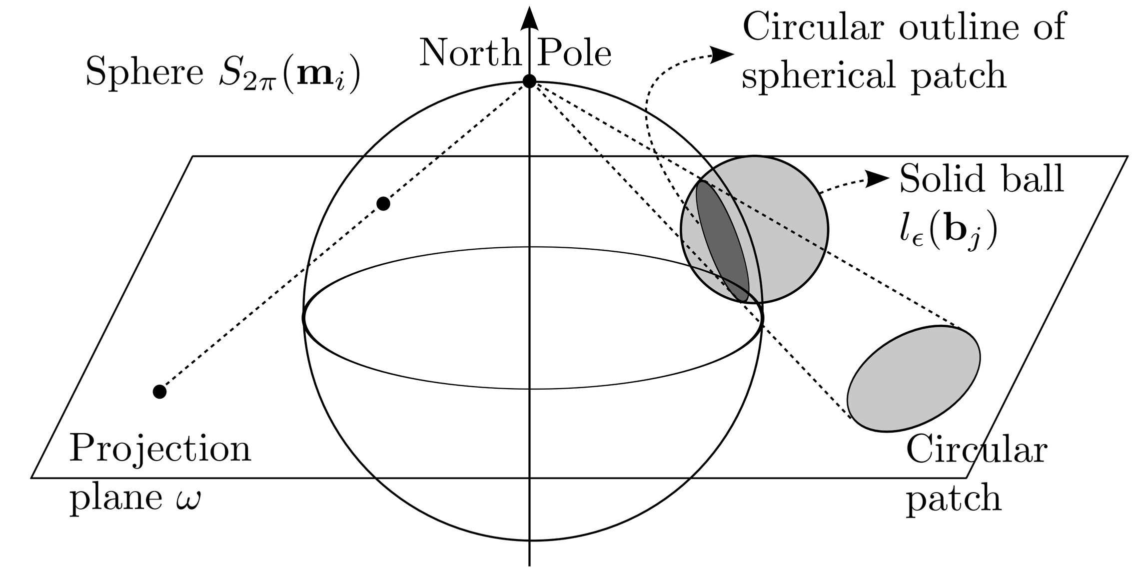
- (Left) Obtains 3D keypoint matches across the input point clouds.
- (Right) Each keypoint match is used to translate the point clouds such that the matched points coincide. If the keypoint match is a true positive, the point clouds can be aligned by just a 3D rotation. Note that our rotation search method uses the original (unmatched) points as input, not the keypoint matches.

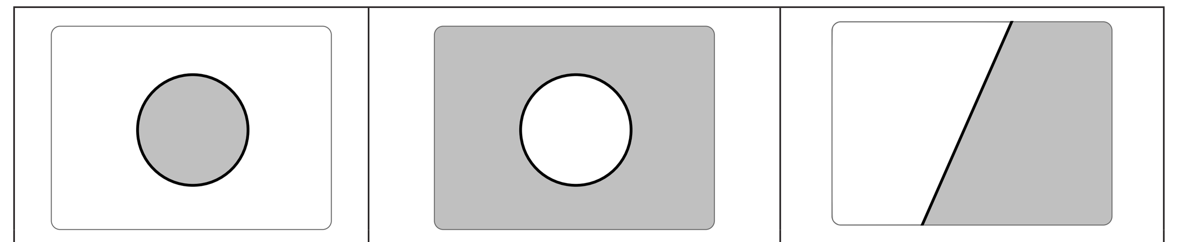
Bound Evaluation

To evaluate $\hat{Q}_{sp}(\mathbb{B})$ efficiently, we need to answer multiple intersection queries like the following quickly:

$$\max_j \lfloor S_{\alpha_{\mathbb{B}}}(\mathbf{R_c}\mathbf{m}_i) \cap l_\epsilon(\mathbf{b}_j) \neq \emptyset \rfloor \qquad (6)$$

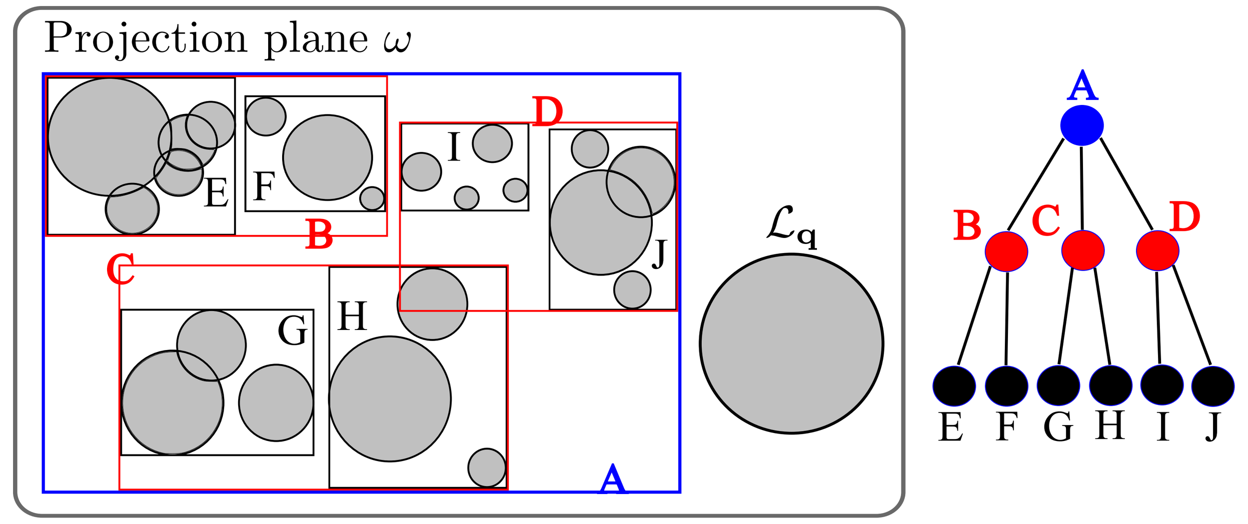
Stereographic projection of spherical patches





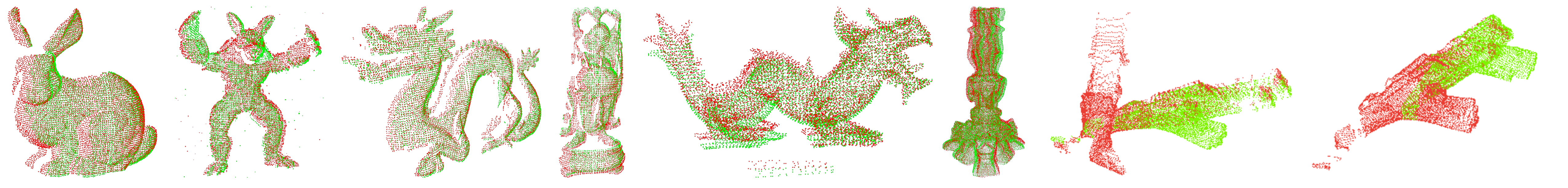
Interior Patch Above example	Exterior Patch Patch contains the pole	Half-Plane The pole lies on the patch circular outline
Indexed in a R-Tree	Indexed in a list	Indexed in a list

To solve an intersection query we first stereographically project $S_{\alpha_{\mathbb{B}}}(\mathbf{R_c}\mathbf{m}_i)$ to obtain the query patch $\mathcal{L}_q$, then check if $\mathcal{L}_q$ intersects any projected patch.



If $\mathcal{L}_q$ overlaps with the minimum bounding rectangle (MBR) of the visited node, then node's children are traversed; at a leaf node, $\mathcal{L}_q$ is simply tested for overlaps with the interior patches contained therein, and if a hit is encountered the query is terminated instantly. If $\mathcal{L}_q$ does not overlap with the MBR of a node, the whole branch can be ignored.

Results on the 6FOD Framework



Experiment setup To registers two point clouds ($\mathcal{V}_1$, $\mathcal{V}_2$), we use ISS3D to detect 3D keypoints across $\mathcal{V}_1$ and $\mathcal{V}_2$. Then, 100 keypoint matches were produced by using the l_2 norm over PFH descriptors.

bnb-M-circ The proposed method.

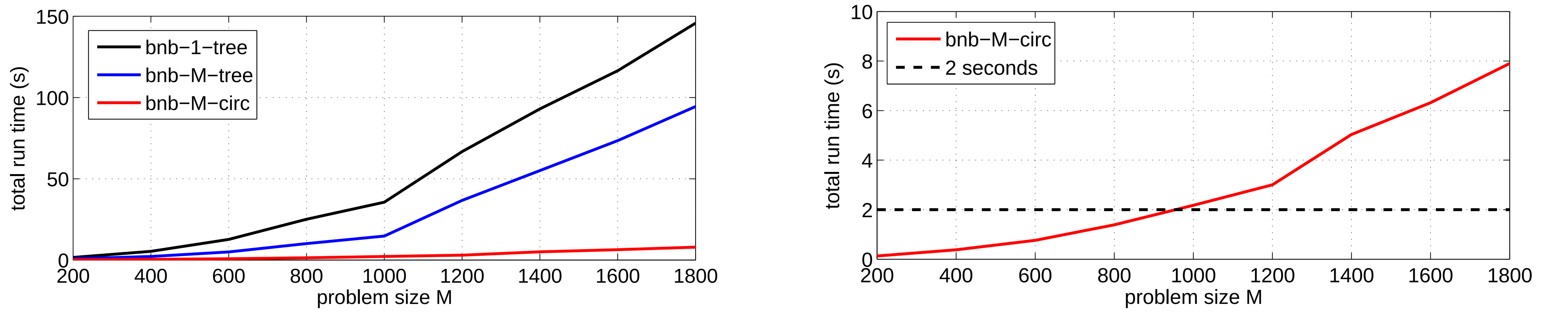
bnb-1-tree Breuel's method using 1 kd-tree.

bnb-M-tree The same as **bnb-M-circ**, but using M kd-trees.

Comparison against RANSAC. Transforms were estimated by three keypoint matches and the quality was taken over all the points. We stopped RANSAC as soon as Q_{glob} equalled or surpassed the Q_{glob} of the BnB methods, or if the number of iterations hit the limit of 50000. The median results from 20 instances of RANSAC are reported.

	Data characteristics				3D rotation search				RANSAC		
Dataset	$\lvert\mathcal{V}_1\rvert$	$\lvert\mathcal{V}_2\rvert$	avg $\lvert\mathcal{M}\rvert$	% inliers	Q_{glob}	**bnb-1-tree** time (s)	**bnb-M-tree** time (s)	**bnb-M-circ** time (s)	med Q_{glob}	med time (s)	med iter
bunny	7055	6742	378.47	43	5377	376.47	128.59	22.11	3292	456.93	49978
armadillo	5619	5483	353.85	15	3243	406.50	152.66	33.09	2795	311.13	42210
dragon	6991	6200	351.45	29	5873	280.34	81.87	13.63	4433	405.88	48316
buddha	5312	5109	372.89	20	4512	331.18	102.45	19.61	3283	348.18	50000
a. dragon	8413	8413	103.16	13	5823	183.89	127.64	20.06	6813	241.96	18755
statuette	44156	44156	205.36	22	18328	163.87	70.83	9.13	14855	2169.25	49448
mineA	7629	7727	187.61	10	3197	2816.35	900.53	218.56	1089	418.54	50000
mineB	7496	5487	330.02	3	4870	6025.66	1978.23	631.13	1265	374.60	50000

Scalability Comparison of Rotation Search Methods



Median run time versus problem size M.

Conclusions

Our rotation search algorithm was shown to be an order of magnitude faster than the original BnB algorithm of Breuel. It also has good performance relative to other methods based on different formulations. Our algorithm was demonstrated to be accurate and efficient in registering partially overlapping point clouds.

References

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[3] J.-C. Bazin and Y. Seo and M. Pollefeys. Globally optimal consensus set maximization through rotation search, In *ACCV '12*