Puzzle-Based Learning

Zbigniew Michalewicz[1]
School of Computer Science, University of Adelaide, Adelaide, SA 5005, Australia
zbyszek@cs.adelaide.edu.au

Matthew Michalewicz
SolveIT Software, Level 12, 90 King William Street, Adelaide, SA 5000, Australia
matthew.michalewicz@solveitsoftware.com

Abstract: *The paper addresses a gap in the educational curriculum for engineers by proposing a new course that aims at getting engineering students to think about how to frame and solve unstructured problems. The idea is to increase the student's mathematical awareness and problem-solving skills by discussing a variety of puzzles. The paper makes an argument that this approach – called Puzzle-Based Learning – is very beneficial for introducing mathematics, critical thinking, and problem-solving skills.*

Introduction

What is missing in most engineering curricula is coursework focused on the development of problem-solving skills. Most engineering students never learn how to think about solving problems in general – throughout their education, they are constrained to concentrate on textbook questions at the back of each chapter. So, without much thinking, they apply the material from each chapter to solve a few problems given at the end of each chapter (why else would a problem be at the end of the chapter?). With this type of approach to "problem solving," it is unsurprising that engineering students are ill prepared for addressing real-world problems. When they finally enter the real world, they suddenly find that problems do not come with instructions or guidebooks.

Young people often have serious difficulties in independent thinking (or problem-solving skills) regardless of the nature of a problem. At the same time educators are interested in teaching "thinking skills" rather than "teaching information and content." The latter approach dominated in the past – whether in engineering, history, physics, geography, or in any other subject. As Alex Fisher wrote in his book, *Critical Thinking*: "*… though many teachers would claim to teach their students 'how to think', most would say that they do this indirectly or implicitly in the course of teaching the content which belongs to their special subject. Increasingly, educators have come to doubt the effectiveness of teaching 'thinking skills' in this way, because most students simply do not pick up the thinking skills in question*." What is worse, engineering students almost never learn how to think about solving problems in general.

Over the past few decades, various people and organizations have attempted to address this educational gap by teaching "thinking skills" based on some structure (e.g. critical thinking, constructive thinking, creative thinking, parallel thinking, vertical thinking, lateral thinking, confrontational and adversarial thinking). However, all these approaches are characterized by a departure from mathematics as they concentrate more on "talking about problems" rather than "solving problems." It is our view that the lack of problem solving skills in general are the consequence of decreasing levels of mathematical sophistication in modern societies (see also the book of John Allen Paulos, *Innumeracy: Mathematical Illiteracy and Its Consequences*). Thus it is necessary, especially for engineers, to connect thinking and problem-solving skills with mathematical awareness as the current approaches are not satisfactory.

[1] Also at Institute of Computer Science, Polish Academy of Sciences, and the Polish-Japanese Institute of Information Technology.

The approach

Hence, we believe that a different approach is needed. To address this gap in the educational curriculum, we have created a new course that is aimed at getting engineering students to *think* about how to frame and solve *unstructured* problems (those that are not encountered at the end of some textbook chapter …). The idea is to increase the student's mathematical awareness and problem solving skills by discussing a variety of *puzzles*. In other words, we believe that the course should be based on the best traditions introduced by Gyorgy Polya and Martin Gardner during the last 60 years. In one of our favourite books, *Entertaining Mathematical Puzzles*, Martin Gardner wrote:

> "*Perhaps in playing with these puzzles you will discover that mathematics is more delightful than you expected. Perhaps this will make you want to study the subject in earnest, or less hesitant about taking up the study of a science for which a knowledge of advanced mathematics will eventually be required*."

Many other mathematicians have expressed similar views. For example, Peter Winkler in his book *Mathematical Puzzles: A Connoisseur's Collection* wrote: "*I have a feeling that understanding and appreciating puzzles, even those with one-of-a-kind solutions, is good for you*."

This new course is the result of many years of experience in educating young engineers, mathematicians, and computer scientists on many levels at many universities in many countries (USA, Mexico, Argentina, New Zealand, Australia, South Korea, Japan, China, Singapore, Poland, Sweden, Germany, Spain, Italy, France, UK). Limited experiments using the puzzle-based learning approach with these students have already produced amazing results: outstanding course evaluations and countless comments that praise the problem-solving orientation of the course. We believe that the main reasons behind most students' enthusiasm for the puzzle-based learning approach are:

1. Puzzles are educational, as they illustrate many useful (and powerful) problem-solving rules in a very *entertaining* way.
2. Puzzles are engaging and thought-provoking.
3. Contrary to many textbook problems, puzzles are not attached to any chapter (as is the case with real-world problems).
4. It is possible to talk about different techniques (e.g. simulation, optimization), disciplines (e.g. probability, statistics), or application areas (e.g. scheduling, finance) and illustrate their significance by discussing a few simple puzzles. At the same time, the students are aware that many conclusions are applicable to the broader context of solving real-world problems.

Further supporting evidence

As a matter of fact, the puzzle-based learning approach has a much longer tradition than just 60 years. The first mathematical puzzles were encountered in Sumerian texts that date back to around 2,500 BC. Yet the best evidence of the puzzle-based learning approach can be found in the works of Alcuin, an English scholar born around AD 732 whose main work was *Problems to Sharpen the Young* – a text which included over 50 puzzles. Some twelve hundred years later, one of his puzzles is still used by countless artificial intelligence textbooks![2]

The first author of this paper is a member of Editorial Board of the *International Journal Teaching Mathematics and Computer Science*. It is clear that new methods of teaching (especially engineers) are sought and experimented with. Further, one of the earlier books by the first author, *How to Solve It: Modern Heuristics*, included a selection of puzzles to illustrate some problem solving activities. Despite the fact that the book aimed at graduate students interested in genetic algorithms, neural

[2] The puzzle is the "river crossing problem": *A man has to take a wolf, a goat, and some cabbage across a river. His rowboat has enough room for the man plus either the wolf or the goat or the cabbage. If he takes the cabbage with him, the wolf will eat the goat. If he takes the wolf, the goat will eat the cabbage. Only when the man is present are the goat and the cabbage safe from their enemies. All the same, the man carries wolf, goat, and cabbage across the river. How has he done it?*

networks, fuzzy systems, and many other traditional and modern techniques, the readers – because of puzzles – got much more than just information of particular techniques. Some comments (still available on www.amazon.com) were:

- "This book teaches you how to *think* of a solution for the problem you face…"
- "…anyone interested in […] human *thinking* should read and understand this book."
- "I used this book in a Master's class on Heuristics (Systems Engineering, University of Virginia) and received the most positive textbook reviews I have seen in my fifteen years of teaching."
- "Most importantly, it does so in a way that no other book I've seen does – it makes it fun and it makes you *think*!"

Importance of Mathematics

Over the years, two primary approaches to problem solving have emerged. One is the *technical* approach (represented in many textbooks), which concentrates on specific problem-solving techniques. The other is the *psychological* approach, which is based on structural thinking – meaning that some structure is imposed on the thinking process during the problem solving activity.

Let's discuss these two approaches in a bit more detail; for that purpose we have selected two popular texts. The first one is *Operations Research: An Introduction* by Hamdy A. Taha, and the other is a book by Edward de Bono, *Six Thinking Hats*. The first book illustrates the technical approach very well, as it is loaded with mathematical techniques for a variety of different problems. On the other hand, the second book presents a particular structured way of thinking. Let us have a closer look at these two books.

Operations Research: An Introduction by Hamdy A. Taha consists of several chapters, each of which relate to a specific problem type. For example, there is a chapter on linear programming, which is a particular technique for solving problems with many variables and where the objective and the values of these variables are expressed as linear expressions. Another chapter of Taha's book discusses a transportation model and its variants, while another presents a series of techniques applicable to network models. There are chapters on goal programming, integer linear programming, dynamic programming, inventory models, forecasting models, etc. Each chapter includes selected references and a problem set. For example, the chapter on inventory models includes the following exercise:

> "*McBurger orders ground meat at the start of each week to cover the week's demand of 300 lb. The fixed cost per order is $20. It costs about $0.03 per lb per day to refrigerate and store the meet. (a) Determine the inventory cost per week of the present ordering policy. (b) Determine the optimal inventory policy that McBurger should use, assuming zero lead time between the placement and receipt of an order. (c) Determine the difference in the cost per week between McBurger's current and optimal ordering policy*."

Clearly, the problem is well-defined and very specific. Earlier parts of the chapter on inventory models discussed, of course, a general inventory model (where the total inventory cost is given as a total of purchasing cost, setup cost, holding cost, and shortage cost) and the classic economic order quantity models. The formula is derived in the chapter to provide the optimum value of the order quantity y (number of units) as a function of setup cost K associated with the placement of an order (in dollars per order), demand rate D (in units per time unit), and holding cost h (in dollars per inventory unit per time unit). The model suggests to order:

$$y = \sqrt{2KD/h}$$

units every y/D time units. This is a good illustration of the technical approach.

It seems that Taha's text is similar to many other texts from disciplines such as engineering, mathematics, finance, and business, in that it has two main characteristics:

(a) the problem types and corresponding techniques are very specific; and
(b) mathematics is used extensively.

However, there is usually no discussion on "how to solve a problem" – the text gives some recipes on how to arrive at solution once the problem has already been reduced to the problem type defined in the text. Students are constrained to concentrate on textbook questions at the back of each chapter, using the information learned in that chapter… So all these specialized texts (whether on probability, statistics, simulations, etc.) that represent the technical approach for problem solving, do not present a problem-solving methodology. They just provide (very useful) information of particular techniques for particular classes of problems.

Let us now turn our attention to the other book, Edward de Bono's *Six Thinking Hats*, which represents the psychological approach. As we have indicated earlier, the book suggests some structure for the thinking process during the problem solving activity. In particular, each of six hats represents a particular function of the thinking process:

White Hat – collection of objective facts and figures

Red Hat – presentation of emotional view

Black Hat – discussion of weaknesses in an idea

Yellow Hat – discussion on benefits of the idea

Green Hat – generation of new ideas

Blue Hat – imposition of control of the whole process

The general idea is that instead of thinking simultaneously along many directions, a thinker should do one thing at the time. Edward de Bono explains it very clearly:

> "*The main difficulty of thinking is confusion. We try to do too much at once. Emotions, information, logic, hope and creativity all crowd in on us. It is like juggling with too many balls.*
>
> *What I am putting forward in this book is a very simple concept which allows a thinker to do one thing at a time. He or she becomes able to separate emotion from logic, creativity from information, and so on. The concept is that of the six thinking hats. Putting on any one of these hats defines a certain type of thinking*."

It seems that *Six Thinking Hats* is characterized by two facts (as are many other texts on thinking processes, which includes texts on critical thinking, constructive thinking, creative thinking, parallel thinking, vertical thinking, lateral thinking, confrontational, and adversarial thinking, to name a few):
(a) the problem types and corresponding "techniques" are *not* very specific. The approach is very general and it applies to most problems (as opposed to specific problem types); and
(b) the approach is mathematics-free.

Indeed, the examples given in the *Six Thinking Hats* vary from house selling activities, through advertising and marketing issues, to pricing products. Further, the mathematics is non-existent despite the fact that some problems may require more precise mathematics. There is no question that the approach proposed by Edward de Bono is very useful and that many corporations benefited from the methodology of *Six Thinking Hats*. On the other hand, the rejection of mathematics in *Six Thinking Hats* expresses itself even in the author's statements, such as: "*In a simple experiment with three hundred senior public servants, the introduction of the Six Hats method increased thinking productivity by 493 percent*." Well, this is very impressive, but any person with any "critical thinking" skills (or some fancy for precision) may ask for clarifications:

> *What is the definition of productivity (especially in cases of senior public servants) and how such productivity – and improvement in productivity – has been measured?*

Indeed, these are very important questions. In the case of the public servants, did three hundred employees fill out forms that evaluated their (increased) productivity? If so, then this can be compared to an example provided by Darrell Huff in his book *How to Lie with Statistics*. The *San Francisco Chronicle* published an article entitled "*British He's Bathe More Than She's*" and the story supported the title with the following facts (based on hot-water survey of 6,000 representative British homes).

"The British male over 5 years of age soaks himself in a hot tub on an average of 1.7 times a week in the winter and 2.1 times in the summer. British women average 1.5 baths a week in the winter and 2.0 in the summer." Darrell Huff, discussing this case, made an excellent (and very important) observation. He wrote:

> "*…the major weakness is that the subject has been changed. What the Ministry really found out is how often these people said they bathed, not how often they did so. When a subject is an intimate as this one is, with the British bath-taking tradition involved, saying and doing may not be the same after all.*"

It seems that the same argument can be applied to the public servants…Most likely, their productivity was measured in hours (i.e. the shorter the time to make a decision, the better). Edward de Bono explains:

> "*A major corporation used to spend twenty days on their multinational project team discussion. Using the parallel thinking of the Six Hats method, the discussions can now take as little as two days*."

However, if this was that case, then it seems that the improvement measure is concerned with one dimension only, as *the quality* of the decisions reached is ignored and not measured! One can argue that we should not care so much whether the problem solving process took x or y hours, as *the quality* of solution is the most important aspect.

There is an excellent book (on science and education, one can say) by Eliyahu M. Goldratt and Jeff Cox, *The Goal*. The book describes the struggle of a plant manager who tries to improve factory performance. He worries about productivity, excess inventories, throughput, balancing capacities, and many other measurements. Only with the help of a consultant does he realize that there is only one goal and one measurement: "*The goal of a manufacturing organization is to make money and everything else we do is means to achieve the goal*." Similarly, in the problem solving process there is only one goal: To find the best possible solution. Of course, very often there is a trade-off between the time needed to find a solution and the quality of the solution (this is often discussed in Computer Science courses on analysis of algorithms), but is seems that the *Six Thinking Hats* method is concerned with only the secondary aspect of problem solving: time efficiency. Precise evaluation of the solution is of lesser importance.

Thus the psychological approach looks like the *opposite extreme* of the technical approach in the spectrum of problem-solving methodologies, as the former focuses on organizational issues of "thinking" for general problems, rather than specific techniques on how to arrive at a solution. Furthermore, the psychological approach uses natural language to describe its mechanisms, whereas the technical approach uses mathematics as a problem solving language.

Which of these two approaches (technical versus psychological) should be used in the real world? Well, each of these two approaches has a crowd of enthusiasts and supporters; however, it seems that the technical approach is based on the solid fundamentals of science. Even some philosophers and psychologists tend to agree. One of the pearls of wisdom taught by Anthony de Mello in his famous book, *One Minute Wisdom*, was the following observation:

> "*Weeks later, when a visitor asked him*
> *what he taught his disciples, he said,*
> *`To get their priorities right: Better*
> *have the money than calculate it; better*
> *have the experience than define it.'* "

It is easy to extend the above statements (while preserving their spirit) by stating that:

> *It is better to know how to solve problems*
> *than to have the ability to talk about them!*

On the other hand, representatives of the technical approach admit that "*although mathematics is a cornerstone of Operations Research, one should not 'jump' into using mathematical models until simpler approaches have been explored. In some cases, one may encounter a 'commonsense' solution* *through simple observations. Indeed, since the human element invariably affects most decision problems, a study of the psychology of people may be key to solving the problem*" (Hamdy A. Taha, *Operations Research: An Introduction*). These comments are followed by a delightful example, where the problem of slow elevator service in a large office building was solved not by the use of mathematical queuing analysis or simulation, but by installing full-length mirrors at the entrance to the elevators: the complaints disappeared as people were kept occupied watching themselves (and others) while waiting for the elevator!

Clearly, there are many merits in concepts related to critical, vertical, lateral, and other thinking paradigms. However, mathematics – the queen of all sciences – must remain the universal language of problem solvers. Otherwise, as we saw, there is a danger of making imprecise statements, and what is worse, there is a danger of finding – and implementing – poor solutions!

Numerous mathematicians have put a lot of effort into finding a middle ground between the technical and psychological approaches to problem solving. The best known work, without a doubt, is Gyorgy Polya's *How to Solve It*, which stands out as one of the most important contributions to problem-solving literature of the twentieth century. Even now, as we have moved into the new millennium, the book continues to be a favourite among teachers and students for its instructive methods. Other works include *I Hate Mathematics* written by Marylin Burns, which is full of tips and methods for solving problems.

Problem/Project-Based Learning vs. Puzzle-Based Learning

There are other well-established learning methodologies that address some of the above issues; these include problem-based learning and project-based learning (e.g. Blumenfeld et al. 1991, Bransford et al. 1986). Note, however, that the problem- and project-based approaches deal with quite complex situations where usually there is no one clear, unique, or correct way of proceeding. For example, projects may include assignments such as: *Where is the best location for a new airport in our city?* Or: *How can we run an efficient marketing campaign for a new product with a limited budget?* There may not be a single best solution to these problems or projects. Usually, the emphasis in these approaches is in how to deal with the complexity of the problem and how to integrate the use of a wide range of techniques. Furthermore, project-based learning may involve teams of people with perhaps different specialist knowledge. With both problem and project-based learning a major piece of work is conducted under the supervision of an experienced facilitator acting in a mentoring role.

The problem-based learning approach proposed in the 1960s at McMaster University Medical School is an instructional method that challenges students to "learn to learn," working cooperatively in groups to seek solutions to real-world problems. Problem-based learning is aimed at enhancing content knowledge and fostering the development of communication, problem-solving, and self-directed learning skills. It has since been implemented in various undergraduate and graduate programs around the world.

Today the defining characteristics of problem-based learning are:

- Learning is driven by challenging, open-ended problems.
- Students work in small collaborative groups.
- Teachers take on the role of "facilitators" of learning.

Accordingly, students are encouraged to take responsibility for their group and organize and direct the learning process with support from a tutor or instructor.

In other words, problem-based learning is any learning environment in which the problem drives the learning. That is, before students learn some knowledge they are given a problem. The problem is posed so that the students discover that they need to learn some new knowledge before they can solve the problem. Student participation involves hands-on investigative/laboratory activities that develop inquiry and intellectual skills. These activities give students an opportunity to appreciate the spirit of science and promote the understanding of the nature of learning.

The classic example of problem-based learning is the famous “Egg-Drop” experiment which has been a standard in science instruction for many years. In this experiment students are asked to construct some type of container that will keep a raw egg from cracking when dropped from ever-increasing elevations. A number of different groups can be set up to search for ways of approaching this problem. Students will be confronted with some long-standing and resilient misconceptions concerning free-fall (for instance, that heavy objects fall to the earth quicker/slower than lighter objects). By encouraging experimentation and communication of their results, some students may see the need to use mathematics in their approach to this problem – however, many students would stay with intuitive solutions.

Students may come to value the notion of a prototype as they take part in the design process, and their investment in the project should increase accordingly. The “solving” of this project can be either a group or individual accomplishment depending on how the instructor wishes the dynamics of the class to develop.

In a complementary contrast to problem-based learning, puzzles tend to be at the end of the spectrum. They usually appear to be deceptively simple. Usually they have a single correct answer. An important part of completing a puzzle is understanding what you have learnt by solving the puzzle and how you would apply this knowledge to similar puzzles.

So puzzle-based learning offers very different intellectual feast for the “Egg-Drop” experiment. Suppose you wish to know which floors in a high building are safe to drop eggs in a special container from and which floors will cause the eggs to break on landing. We can eliminate chance and possible differences between different eggs (e.g. one egg breaks when dropped from the 7^{th} floor and another egg survives a drop from the 20^{th} floor) by making a few (reasonable!) assumptions, e.g. that an egg that survives a drop can be used again (no harm is done and the egg is not weaker), that a broken egg can not be used again for any experiment, that the effect of a fall is the same for all eggs, that if an egg breaks when dropped from some floor, it would break also if dropped from a higher floor, and that if an egg survives a fall when dropped from some floor, it would survive also if dropped from a lower floor. Obviously, if only one egg is available for experimentation to determine the first egg-breaking floor, we have to start with dropping the egg from the first floor. If it breaks, we know the answer. If it survives, we drop it from the second floor. And we continue upward until the egg breaks. Clearly, the worst-case scenario would require as many drops as the number of floors in the building... Now, the challenge starts when we have two available eggs. What is the least number of egg drops required to determine the egg-breaking floor?

To solve this problem, no laboratory is required: just basic problem-solving skills, plus the ability to add and subtract numbers! We believe that this puzzle-based version of the “Egg-Drop” problem is of equal intellectual value and complements the original “Egg-Drop” experiment offered by the problem-based learning approach.

Since problem-based learning starts with a problem to be solved, students working in a problem-based learning environment should be skilled in problem solving or critical thinking or "thinking on your feet" (as opposed to rote recall). Many educators believe that some qualifying examinations – in which the problem solving (thinking skills) of the candidates are tested – should be conducted before the candidates are admitted. In the McMaster University Medical School, one of five criteria for admission is a test of the candidates’ problem solving skills. Unfortunately, many universities introduce problem-based learning courses without either pre-screening or developing their students’ skill in problem solving... So a puzzle-based learning course (or unit) may fit very well as a prerequisite for later problem-based learning activities.

Current state

The new course (which aims at getting engineering students to *think* about how to frame and solve *unstructured* problems) has been approved by the University of Adelaide for Faculty of Engineering, Computer Science, and Mathematics. The course will be offered in two versions: (a) full-semester course and (b) a unit within general course (e.g. Introduction to Engineering). Many other universities are in the preliminary phase of introducing such a course. All teaching materials (power point slides, assignments, etc.) are being prepared. The new textbook (*Puzzle-Based Learning: Introduction to Critical Thinking, Mathematics, and Problem Solving*) will be available from July 2008. Please, contact one of the authors if you think your organization might be interested in introducing this course.

We believe that besides being a lot of fun, the puzzle-based learning approach will also do a remarkable job of convincing engineering students that (a) science is useful and interesting, (b) the basic courses they take are relevant, (c) mathematics is not *that* scary (no need to hate it!), and (d) it is worthwhile to stay in school, get a degree, and move into the real world which is loaded with interesting problems (problems perceived as real-world puzzles…). These points are important, as most students are unclear about the significance of the topics covered during their studies. Oftentimes, they do not see a connection between the topics taught (e.g. linear algebra) and real-world problems, and they lose interest with predictable outcomes.

References

Blumenfeld, P. C., Soloway, E., Marx, R. W., Krajcik, J. S., Guzdial, M., & Palincsar, A. (1991). Motivating project-based learning: Sustaining the doing, supporting the learning. *Educational Psychologist, 26* (3 & 4), 369-398.

Bransford, J. D., Sherwood, R. S., Vye, N. J., & Rieser, J. (1986). Teaching thinking and problem solving: Research foundations. *American Psychologist, 41*, 1078-1089.

Burns, M. (1975). *I Hate Mathematics*, New York: Little, Brown, and Company.

de Bono, E. (1999). *Six Thinking Hats*, New York: MICA Management Resources.

de Mello, A. *One Minute Wisdom*, New York: Doubleday.

Fisher, A. (2001). *Critical Thinking: An Introduction*, Cambridge, UK: Cambridge University Press.

Gardner, M. (1994). *My Best Mathematical and Logic Puzzles*, New York: Dover Publications.

Gardner, M. (1961). *Entertaining Mathematical Puzzles*, New York: Dover Publications.

Goldratt, E.M. & Cox, J. (2002). *The Goal*, 2nd edition, Aldershot, Hampshire: Gower Publishing.

Huff, D. (1993). *How to Lie with Statistics*, New York: W. W. Norton.

Michalewicz, Z. & Fogel, D. (2004). *How to Solve It: Modern Heuristics*, 2nd edition, Berlin: Springer.

Michalewicz, Z. & Michalewicz, M. (2008). *Puzzle-Based Learning: Introduction to Critical Thinking, Mathematics, and Problem Solving*, Melbourne: Hybrid Publishers.

Paulos, J.A. *Innumeracy: Mathematical Illiteracy and Its Consequences*, New York: Hill and Wang.

Polya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method*, Princeton: Princeton University Press.

Taha, H.A. (2003). *Operations Research: An Introduction*, 7th edition, Upper Saddle River: Pearson Education.

Winkler, P. (2004). *Mathematical Puzzles: A Connoisseur's Collection*, Wellesley: A.K. Peters.

Copyright statement