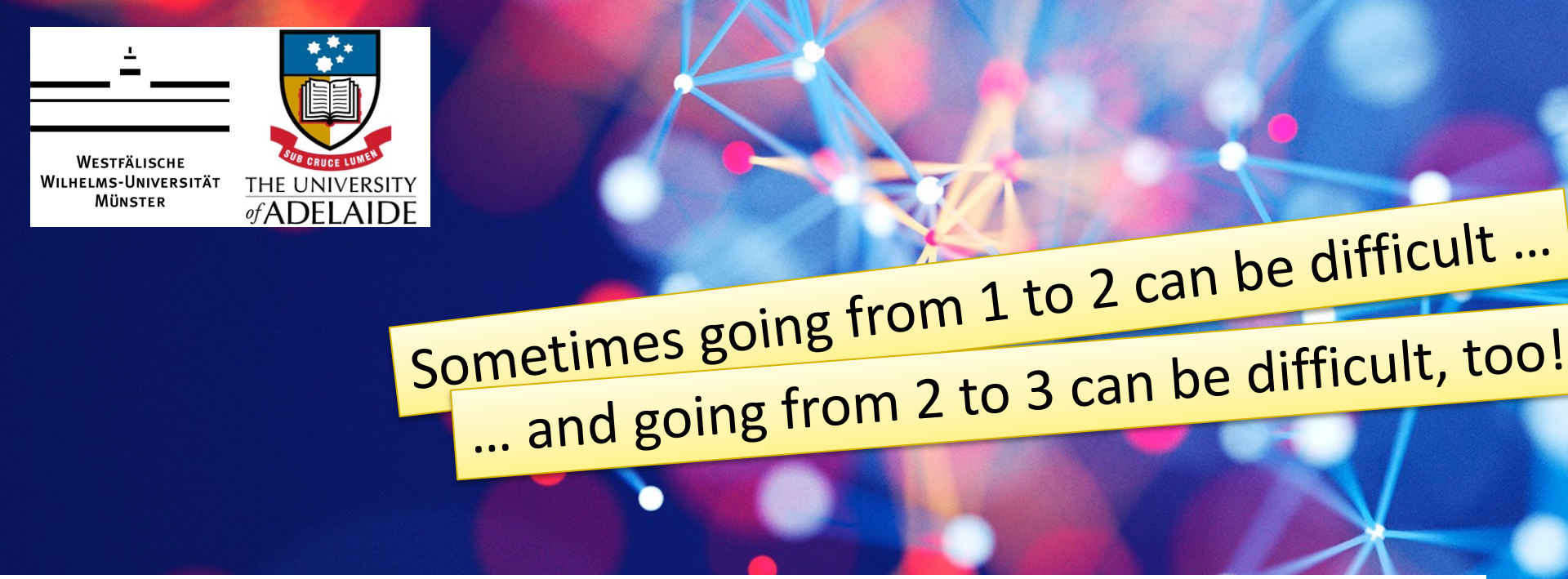

Generating Instances with Performance Differences for More Than Just Two Algorithms

Jakob Bossek, Markus Wagner

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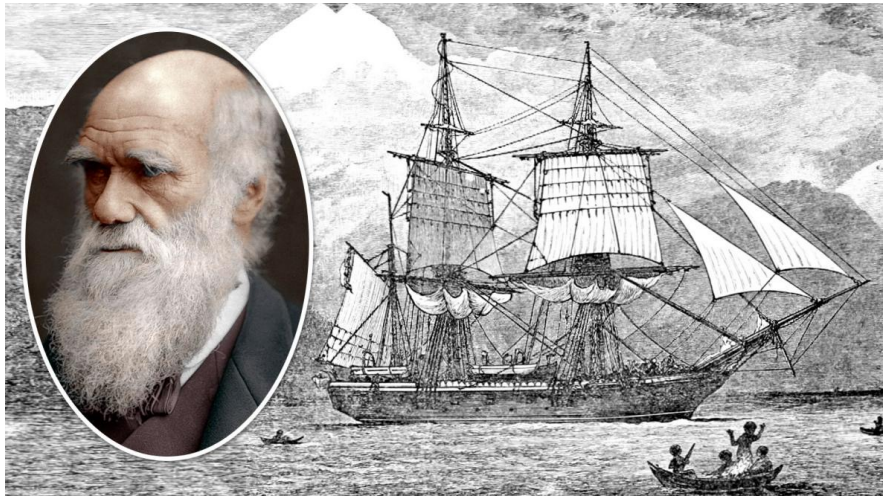
Code at https://github.com/jakobbossek/GECCO2021-ECPERM-ttp-evolving

200 years ago, Charles Darwin



Galápagos Islands

Islas Galápagos

4.6 ★★★★½ 2,344 reviews

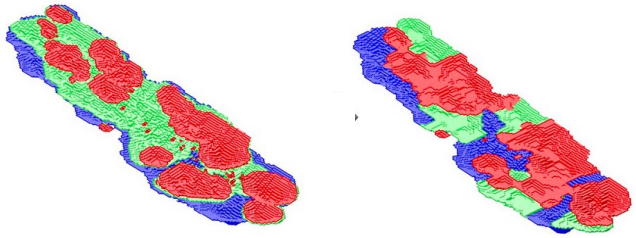
Archipelago

Four of Darwin's finches, clockwise (from top left): *Geospiza magnirostris*, *Geospiza fortis*, *Certhidea fusca*, *Camarhynchus parvulus*

Each species is doing well in its niche ➔ “equally good”

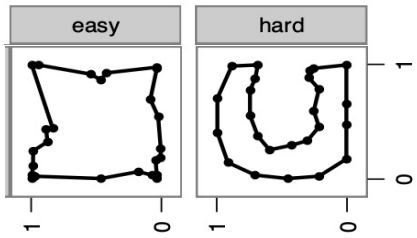
Diversity nowadays: show alternatives

Mine planning:
optimize w.r.t. known objectives (money, time, …) but then show alternative plans (e.g. sequences, …)



Inspirational image generation:
optimize quality & similarity (to a seed), but be diverse (search here via the latent space)

Algorithm understanding / algorithm tuning / algorithm portfolios / …:
generate X instances (*diverse* w.r.t. Y features) on which Z algorithms perform as *differently* as possible



Algorithm footprint

[Background 1/4] Performance Diversity of instances for the Travelling Salesperson Problem

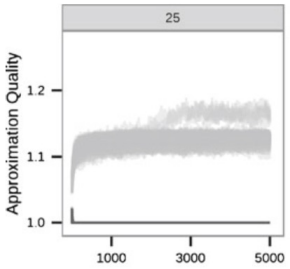
fundamental combinatorial problem: find the shortest tour across n cities

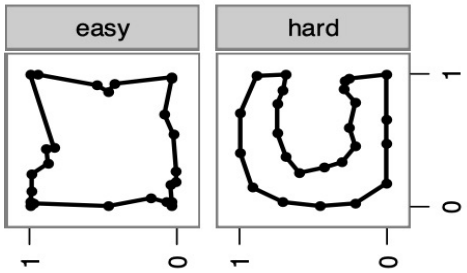
➔ In ~2010: we want to construct a set of TSP instances on which the performance of one algorithm varies

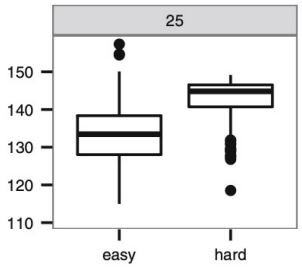
Examples:

- Diverse set where a certain algorithm is performing badly (high approximation ratio): $\alpha(I) = \frac{A(I)}{OPT\ (I)}$
- Diverse set where two solvers A and B are performing differently (again: use performance ratios): $\alpha(I) = \frac{A(I)}{B(I)}$

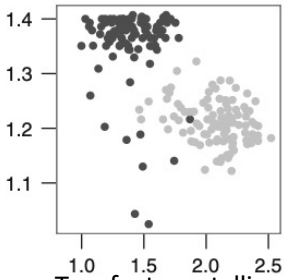
Here:
25 cities







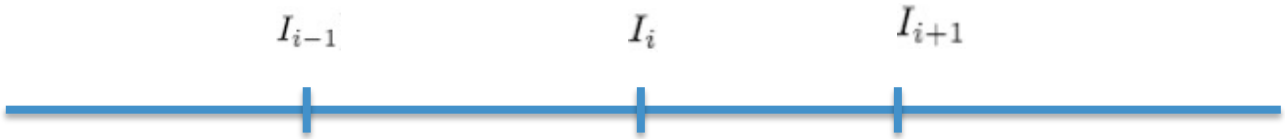
Mean distance between cities in the optimal tour



Two features telling easy and hard apart

[Background 2/4] Single-feature diversity measure
(also shows: going from 2 to 3+ can sometimes be challenging)

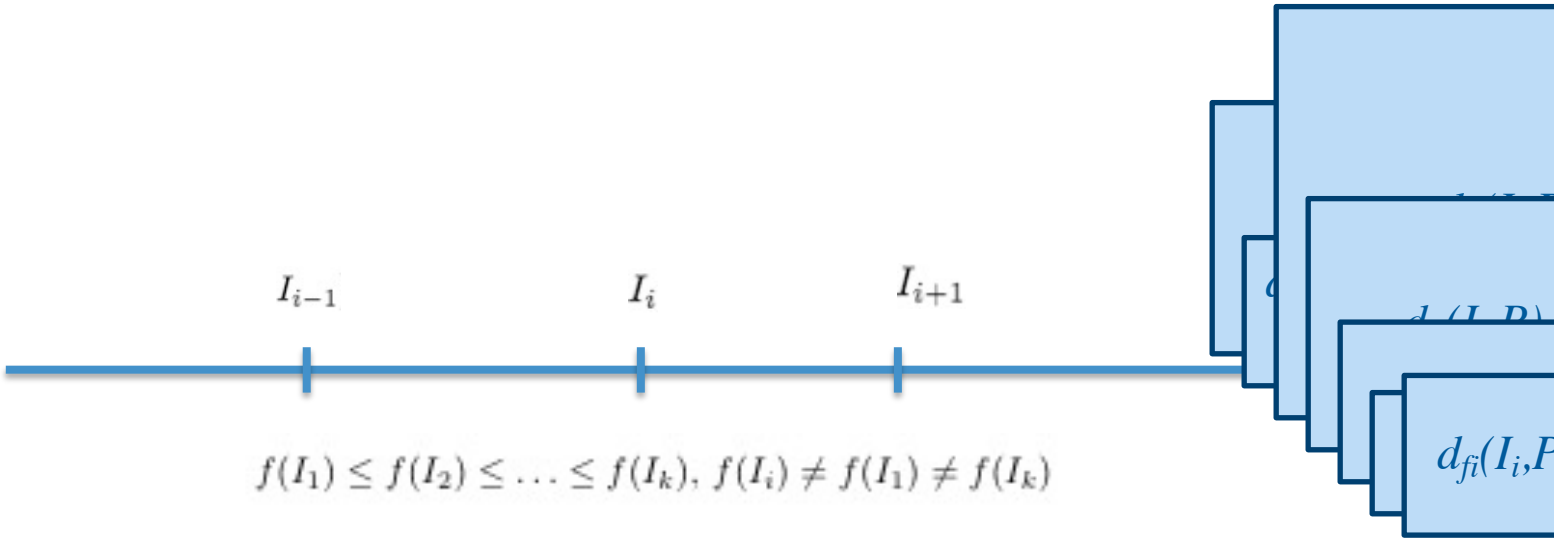
Feature f:



$$f(I_1) \leq f(I_2) \leq \ldots \leq f(I_k),\ f(I_i) \neq f(I_1) \neq f(I_k)$$

[Background 2/4] Single-feature diversity measure
(also shows: going from 2 to 3+ can sometimes be challenging)

Feature f:



"Diversity" of a single solution:

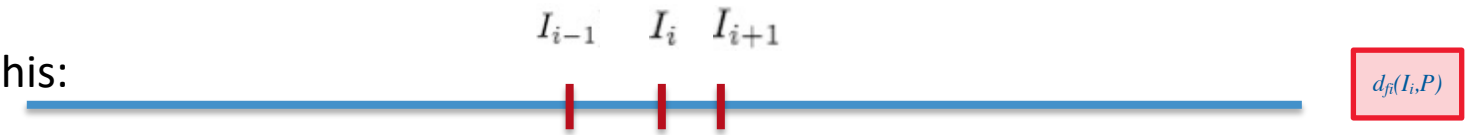
$$d_{f_i}(I_i, P) = (f(I_i) - f(I_{i-1})) \times (f(I_{i+1}) - f(I_i))$$

Diversity of a population:

$$d'(I, P) = \sum_{i=1}^{k} (w_i \times d_{f_i}(I, P))$$

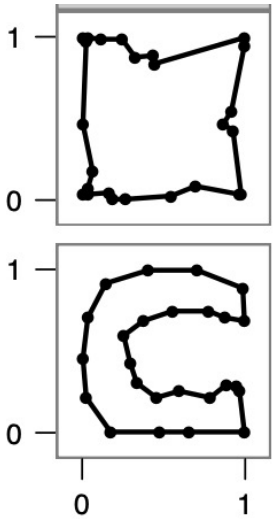
Maximum: if solutions are equally spaced out, as this is then the sum of squares

Compare with this:



EA for evolving several diverse instances for the Traveling Salesperson Problem

(Gao, Nallaperuma, Neumann (PPSN 16))



d(I,P) is the diversity contribution of instance I to the population P.
Diversity here: might be in difficulty, structure, feature, …

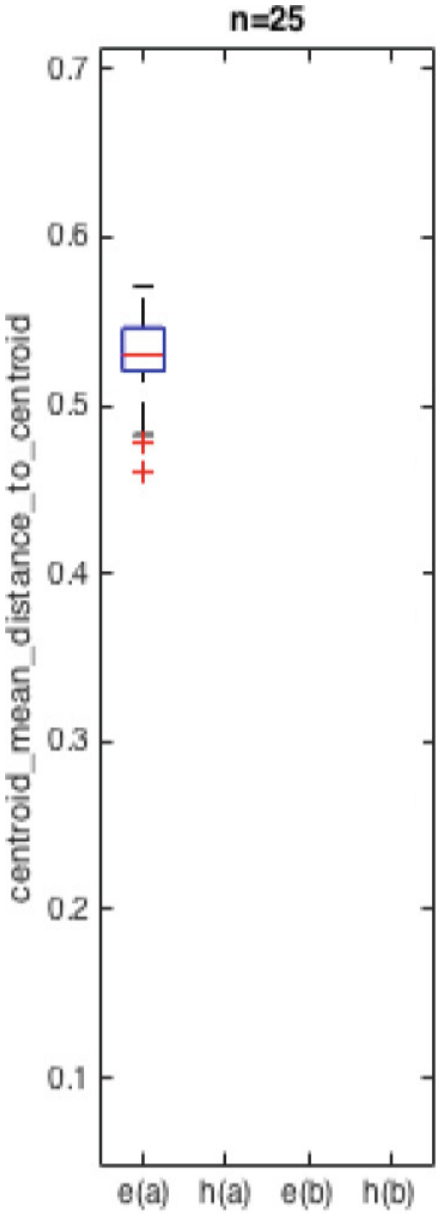
Let I be an individual (tour).

Algorithm 1. $(\mu + \lambda)$-EA_D

1. Initialize the population P with μ TSP instances of approximation ratio at least α_h.
2. Let $C \subseteq P$ where $|C| = \lambda$.
3. For each $I \in C$, produce an offspring I' of I by mutation. If $\alpha_A(I') \geqslant \alpha_h$, add I' to P.
4. While $|P| > \mu$, remove an individual $I = \arg\min_{J \in P} d(J, P)$ uniformly at random.
5. Repeat step 2 to 4 until termination criterion is reached.

Reminder: **α** is used here just as a quality constraint,
and survivor selection does not consider it, but only the diversity (contribution)

[Background 3/4] Features of "easy/hard" TSP instances for ***2-opt*** (with and without diversity optimization)



Feature values of evolved instances:
From left to right:

1. Easy instances / only using **α**
2. Hard instances / only using **α**

} Conclude that this feature is important for instance difficulty (you might expect training an algorithm selector on this data)

3. Easy instances / feature diversity (**α** as quality constraint)
4. Hard instances / feature diversity (**α** as quality constraint)

Works for other features, too… but not for all.

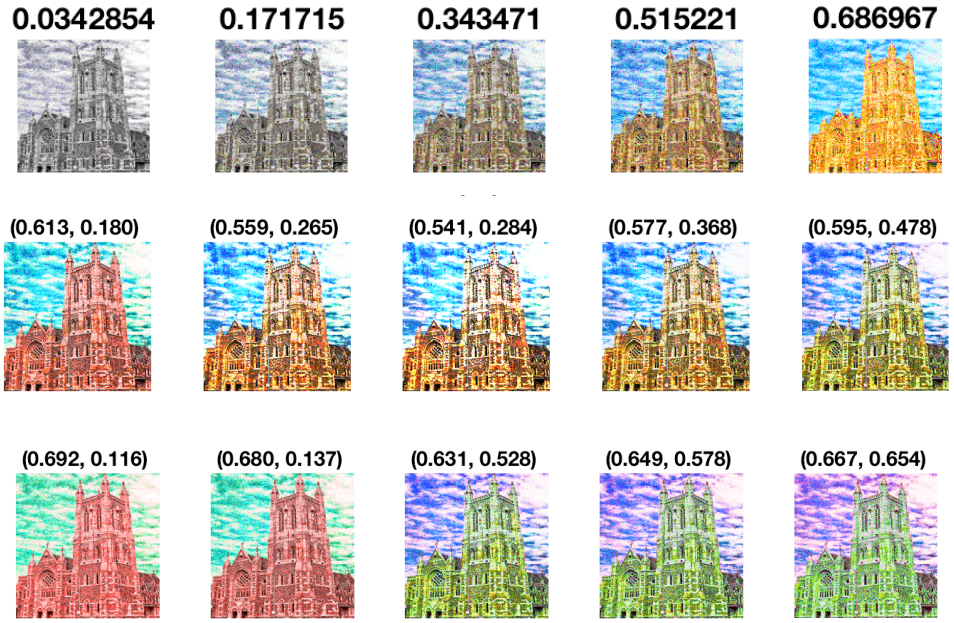
Long story short (here): old conclusions drawn re importance (for difficulty) invalidated by including feature diversity

[Background 4/4] Multi-feature diversity measure

(also shows: going from 1 to 2+ can sometimes be challenging)

- ***Task:*** evolve a diverse set of images that are close to a given image.
- Close means: RMSE to given image is less than 10.

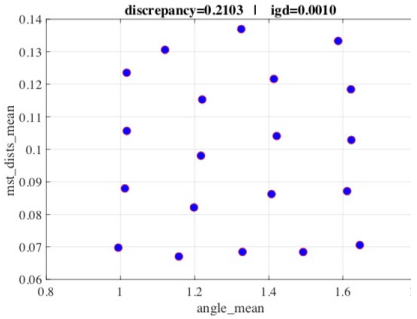
On the right: either ***1 feature*** as target, or ***a linear combination of two features***



→ How to move to 2+ features in a less biased way? Related: what does "diversity" mean in the 2+D space?

[Background 4/4] Multi-feature diversity measure

(also shows: going from 1 to 2+ can sometimes be challenging)



- Feature diversity w.r.t. 2+ features (GECCO'19 BPA nomination)
  ➔ going from 1 to 2+ features has been challenging without favouring one feature (or value range or linear combination or …) over another

Besides all of this: diversity-focused research is a hot field:

- Lots of theoretical runtime analyses by Frank Neumann et al. (GECCO'21 BPA nomination)
- More material: IEEE CEC 2021 tutorial
  https://cs.adelaide.edu.au/~optlog/EvolutionaryDiversityOptimisationTutorialCEC2021.php

➔ ***The OPEN QUESTION this present talk aims to answer:***

How to evolve instances on which >2 algorithms perform as differently as possible?

For example: I_1: $A_1 > A_2 > A_3$ I_2: $A_3 > A_1 > A_2$ I_3: $A_3 > A_2 > A_1$

Approach

What shall our fitness function F be?

Algorithm 1: Outline of instance evolving (1 + 1)-EA.

```
input: Fitness function F
Initialize instance I randomly;
while budget not depleted do                ▷ Often time-limit
    Generate I′ by applying mutation to I;
    if F(I′) ≥ F(I) then
        Replace I with I′;
return I;
```

IDEA 1/3: Pairwise approach

Consider all N(N-1) ordered pairs of algorithms and evolve for each pair in individual runs.

➔

- Easy to formulate with existing tech (see [Background 1/4]):
  maximise $A_1(I)/A_2(I)$, *maximise* $A_3(I)/A_2(I)$, ...
- But: a run ignores all other N-2 algorithms, e.g., a resulting instance might be easy for all of these.

Approach

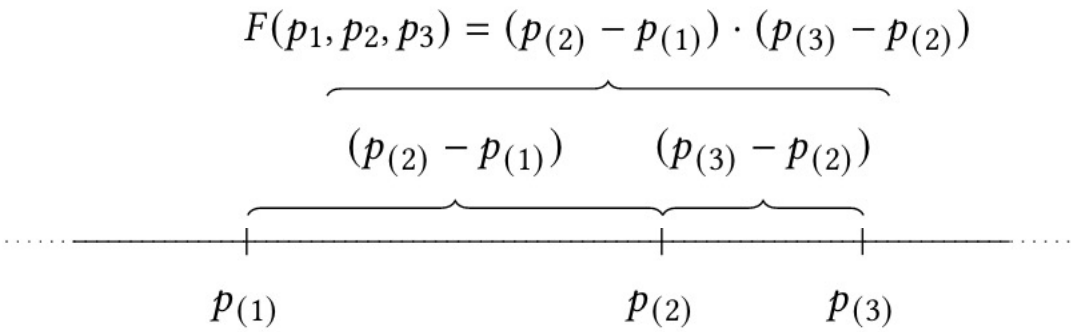
What shall our fitness function F be?

Algorithm 1: Outline of instance evolving (1 + 1)-EA.

```
input: Fitness function F
Initialize instance I randomly;
while budget not depleted do                ▷ Often time-limit
    Generate I' by applying mutation to I;
    if F(I') ≥ F(I) then
        Replace I with I';
return I;
```

IDEA 2/3: No order

Sort performances p_i and then maximise the crowding distance.



➔

- Uses established technology (see [Background 2/4]): *maximise* $F(x_x,p_y,p_z)$
- We need to get lucky to hit a desired permutation.

Approach

What shall our fitness function F be?

Algorithm 1: Outline of instance evolving (1 + 1)-EA.

```
input: Fitness function F
Initialize instance I randomly;
while budget not depleted do                ▷ Often time-limit
    Generate I′ by applying mutation to I;
    if F(I′) ≥ F(I) then
        Replace I with I′;
return I;
```

IDEA 3/3: Explicit ranking

Use a three-tuple to implement two "pull":

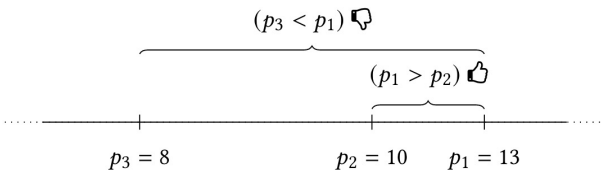
1. Pull: match the desired ranking
2. Pull: maximise the performance difference

Some details:

Let $p_1,\ldots,p_N$ be the performance values of the algorithms, and let π be the desired ranking.

Good directions: $G=\{(i,i+1)|p_{\pi(i)} \geq p_{\pi(i)}\}$

Bad directions: $B=\{(i,i+1)|p_{\pi(i)} < p_{\pi(i)}\}$



→ Maximise lexicographically: $F(p_1,\ldots,p_N; \pi) = (\,|G|, f_B, f_G\,)$

where f_B is the sum of the distances in the bad pairs, and f_G is the sum of the distances in the good pairs.*

Local sensitivity is necessary!

Case Study

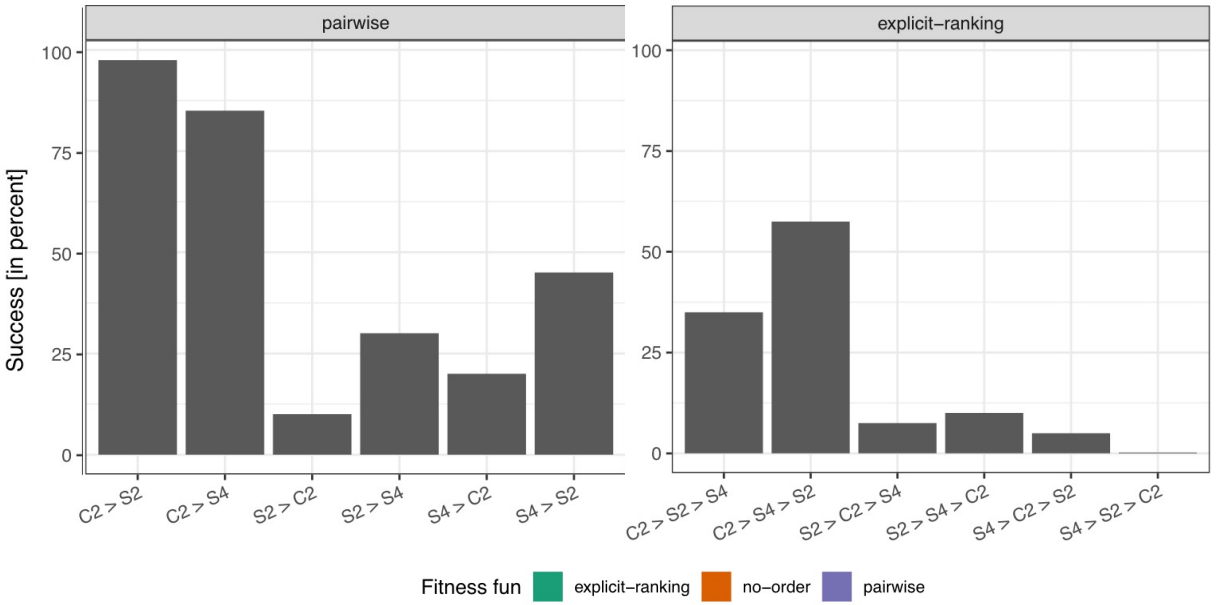
Target problem: Travelling Thief Problem
(partly "find a permutation for the travelling" and partly "find a bitstring for a knapsack")

Target algorithms:
- ***S2*** ("simple"): targets bitstring
- ***S4*** ("simple"): targets permutation
- ***C2*** ("complex"): targets bitstring & permutation alternatingly

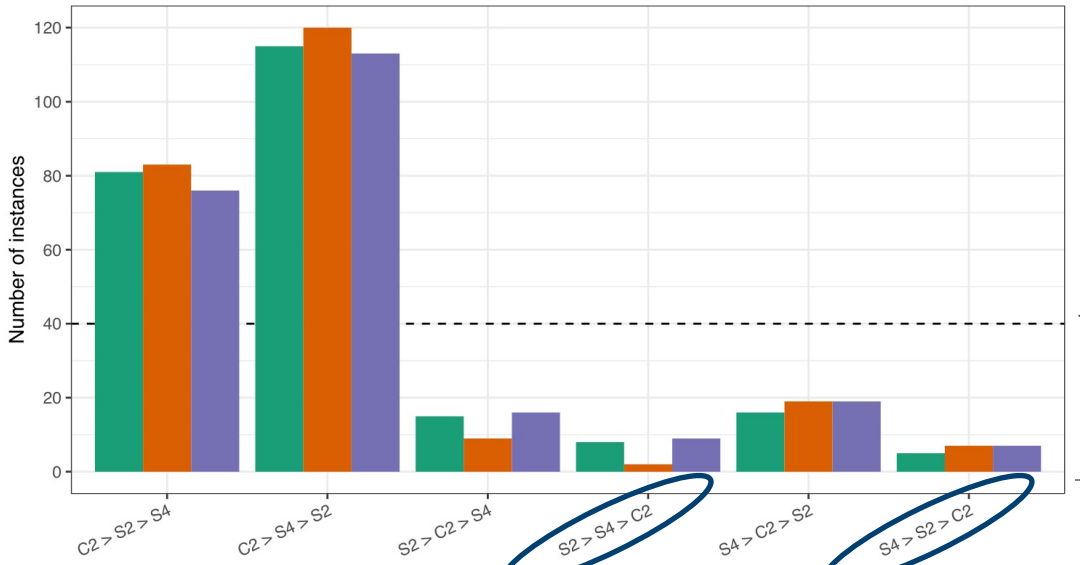
Instance evolution:
- simple (1+1)EA with disruptive operators
- instances with 200 nodes and with 200, ..., 2000 items
- performance on an instance:
  - During evolution: median of 5 runs
  - After evolution: run all three algorithms 30 times

Results: desired vs. actual rankings

% of successful jobs

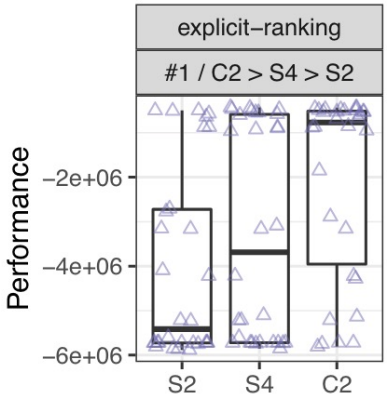


# of instances evolved

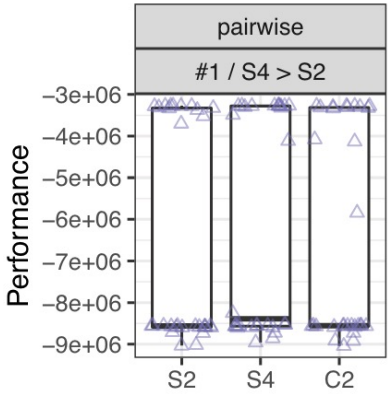


Results: Investigating issues

1) General noise…
(30*3=90 triangles show the raw performance)



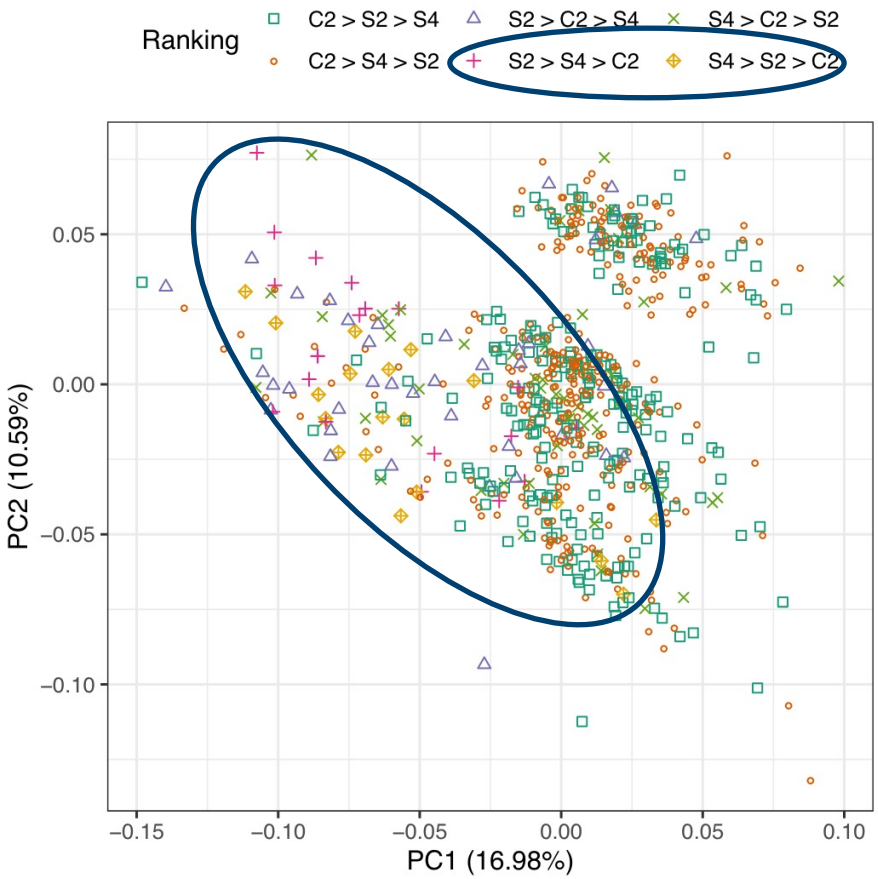
… sometimes bi-modal behaviour



→ Address 1) and 2) by using more repetitions during evolution… but this costs computation time.

2) The complex ***C2*** heuristic is “more powerful” than the other two and tends to dominate.

Results: Properties of instances (1/2)
PCA in the "instance feature"-space



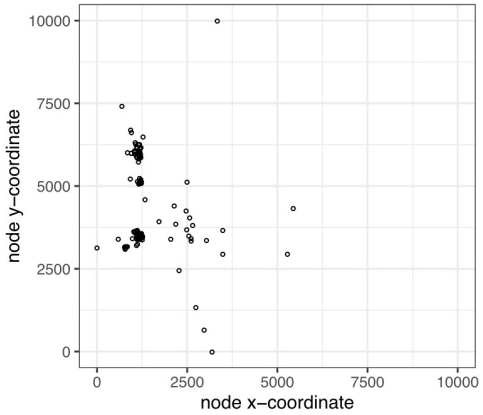
→ For some rankings: instances appear in distinct parts of the "instance feature"-space. But remember [Background 3/4]: we need to be careful when drawing conclusions from such observations, as we have *only* optimised performance rankings.

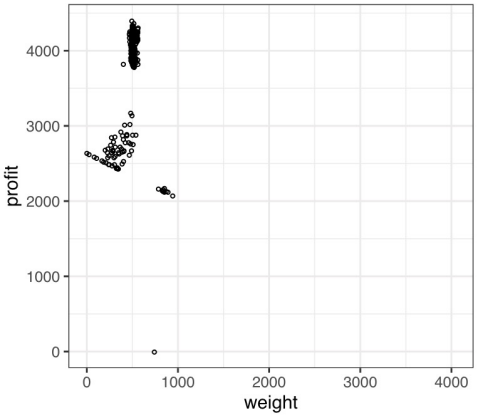
Results: Properties of instances (2/2)
Two Examples

TSP: node coordinates

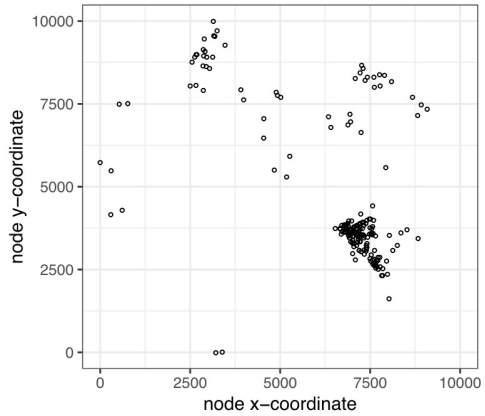
knapsack items: weight & profit

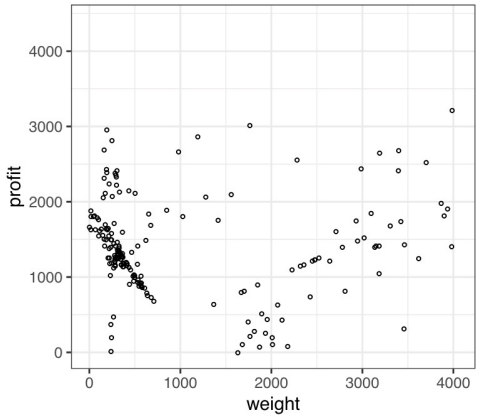
S2 > S4 > C2





C2 > S4 > S2





Summary: Diversity — when optimisation is not the goal

10 years ago: “evolve *one* easy/hard instance for *one* algorithm”

Now: ”evolve sets of solutions (== ‘actual’ solutions, instances, …) that are diverse in multi-dimensional spaces (‘actual’ feature space, performance space, …), and possibly subject to minimum quality constraints”

What is next???

- Optimisation: study custom variation operators [we learned that we have to be disruptive in the encoding space]
- Your solvers: how to change your state-of-the-art solvers to compute diverse sets of solutions (instead of a single one)? [beneficial to the end user, but maybe also during search?]
- Your domains: can ‘diversity’ be helpful to you, if so, why and how?

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