A fitness landscape analysis of the Travelling Thief Problem

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Introduction

Introduction

Objectives:

- Understand the search space structure of the TTP using basic local search heuristics with Fitness Landscape Analysis;
- Distinguish the most impactful non-trivial problem features (exploring the Local Optimal Network representation);

Introduction

Motivation:

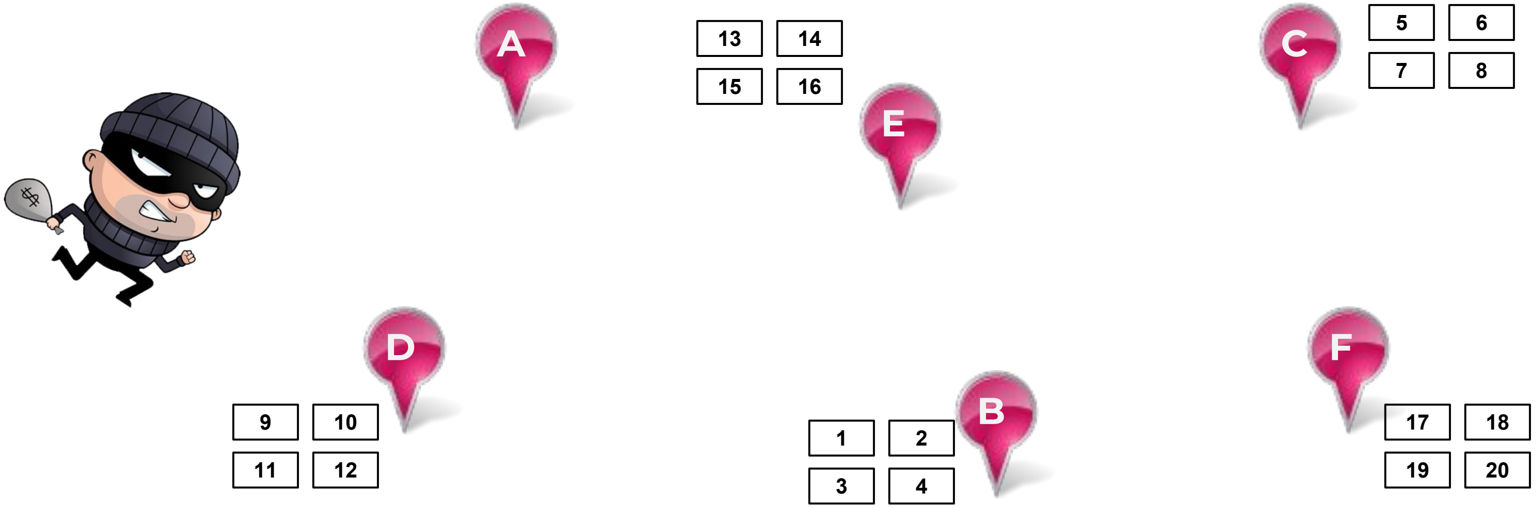
- The TTP -> important aspects found in real-world optimisation problems (composite structure, interdependencies,...);
- Only few studies have been conducted to understand the TTP complexity;
- LONs -> useful representation of the search space of combinatorial (graph theory);
- LONs -> characteristics correlate with the performance of algorithms.

Background

Background

The Traveling Thief Problem:

<<Given a set of items dispersed among a set of cities, a thief with his rented knapsack should visit all of them, only once for each, and pick up some items. What is the **best** path and picking plan to adopt to achieve the best benefits ?>>*

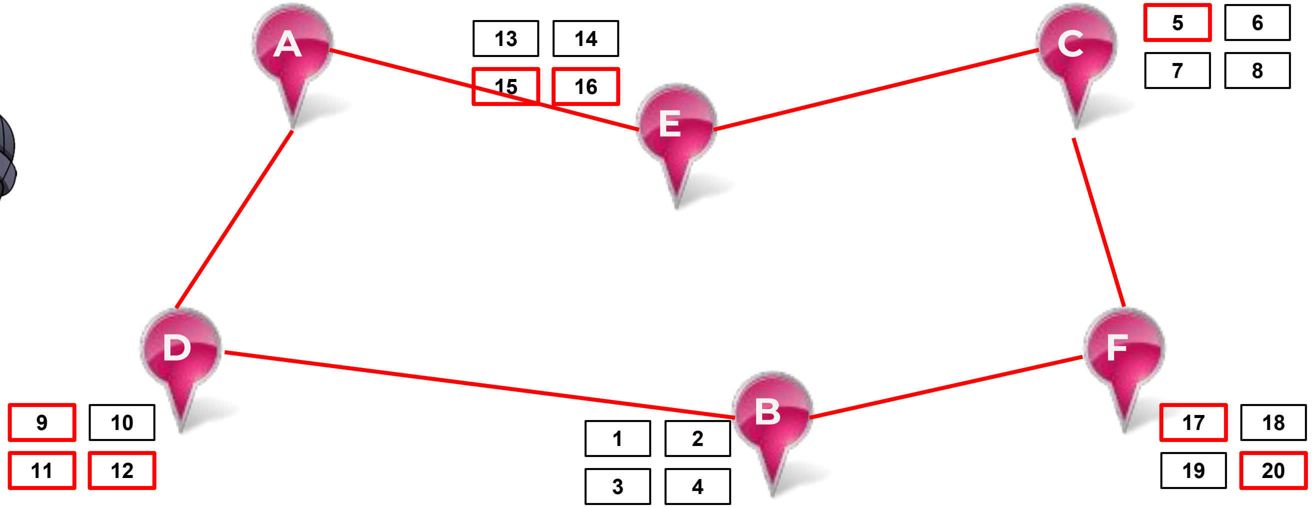


Background

The Traveling Thief Problem:

A TTP solution is represented with two components:

1. The path (eg. ***x***={A, E, C, F, B, D, A})
2. The picking plan (eg. ***y***={15, 16, 5, 17, 20, 9, 11, 12})



Background

The Traveling Thief Problem parameters**:**

- ***W***: The Knapsack capacity
- ***R***: The renting rate
- v_{max}/v_{min}: Maximum/Minimum Velocity

Maximize the total gain:

$$G(x ; y) = total\_items\_value(y) - R * travel\_time(x ; y)$$

The more the knapsack gets heavier, the more the thief becomes slower:

$$current\_velocity = v_{max} - current\_weight * (v_{max} - v_{min}) / W$$

Background

Fitness Landscapes:

*A graph **G**=(**N**,**E**) where nodes represent solutions, and edges represent the existence of a neighbourhood relation given a move operator.*

- ⚠ Defining the neighbourhood matrix for **N** can be a very expensive.
- ⚠ Hard to extract useful information about the search landscape from **G**.

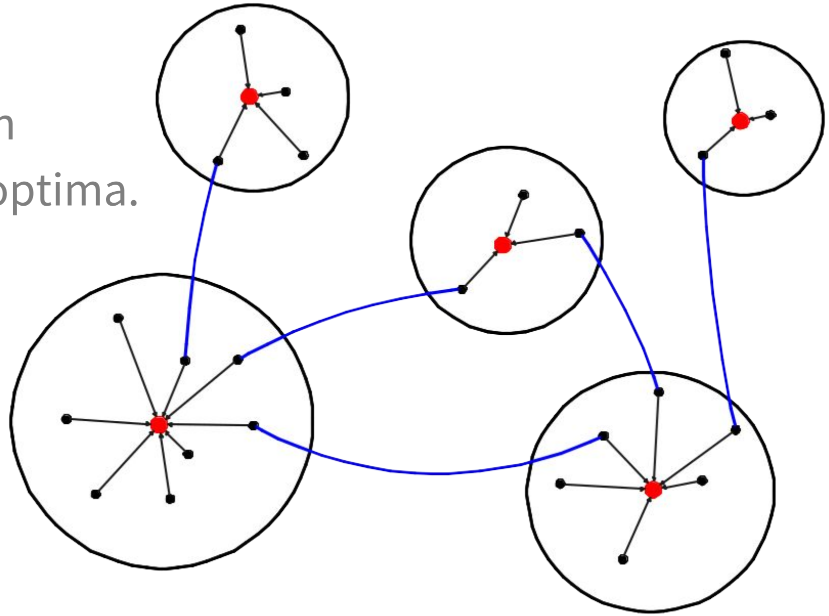
Background

Local Optima Networks:

A simplified landscape representation...

- ✓ Nodes: Local optima / Basins of attraction
- ✓ Edges: Connectivities between the local optima.

Two basins of attraction are connected if at least one solution within a basin has a neighbour solution within the other given a defined move operator.



Background

Local Optima Networks:

- A simplified landscape representation…
- Provides a very useful representation of the search space
- Exploit data by using metrics and indices from graph theory

Environment Settings

Environment Settings

Local Search Heuristics:

- Embedded neighbourhood structure
  - Generates a problem specific neighbourhood function
  - Maintains homogeneity of the TTP solutions

Algorithm 1 A basic local search heuristic framework for the TTP

$s \leftarrow$ initial solution
while there is an improvement **do**
  for each $s^* \mathcal{N}_{TSP}(s)$ **do**
    for each $s^{**} \mathcal{N}_{KP}(s*)$ **do**
      if $F(s^{**}) > F(s)$ **then**
        $s \leftarrow s^{**}$
      end if
    end for
  end for
end while

Environment Settings

Local Search Heuristics:

Two local search variants:

1. ***J2B***: 2-OPT move
2. ***JIB***: Insertion move

} + One-bit-flip operator

Algorithm 1 A basic local search heuristic framework for the TTP

$s \leftarrow$ initial solution
while there is an improvement **do**
  for each $s^* \mathcal{N}_{TSP}(s)$ **do** ← 2-OPT / Insertion
    for each $s^{**} \mathcal{N}_{KP}(s*)$ **do** ← one-bit-flip
      if $F(s^{**}) > F(s)$ **then** ← keep the best in the entire N_{KP} neighborhood
        $s \leftarrow s^{**}$
      end if
    end for
  end for
end while

Environment Settings

- **TTP classification and parameters**
  - Number of cities (*n*);
  - Item Factor (*F*);
  - Profit-value correlation (*T*);
  - Knapsack capacity class (*C*);
- **Instance Generation**
  - 27 classes of the TTP are considered;
  - For each class, 100 samples are generated;

1. uncorrelated (unc)
2. uncorrelated with similar weight (usw)
3. bounded strongly correlated (bsc)

$$W = \frac{C}{11} \sum_{x=2}^{n} \sum_{y=1}^{F} w_{xy}$$

Environment Settings

How we conduct our experiments to achieve the objectives?

1 - Propose a problem classification based on knapsack capacity and the profit-weight correlation;

2 - Create a large set of enumerable TTP instances;

3- Generate a LON for each instance using two hill climbing variants;

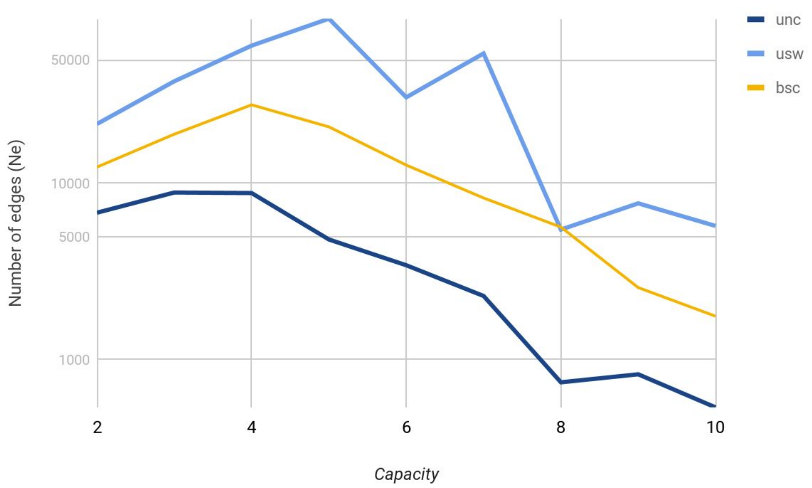
4- Explore/exploit LONs using specific measures.

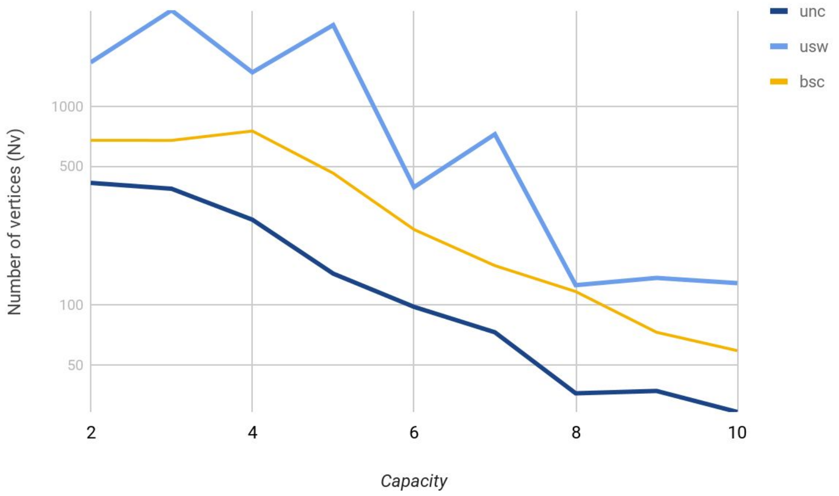
Results & Analysis

Topological properties of LONs

Mean number of vertices ($\overline{n_v}$) & edges ($\overline{n_e}$):

- $\overline{n_v}$ & $\overline{n_e}$ decrease by increasing the knapsack capacity.
- → **hardness of search decreases when the knapsack capacity increases**

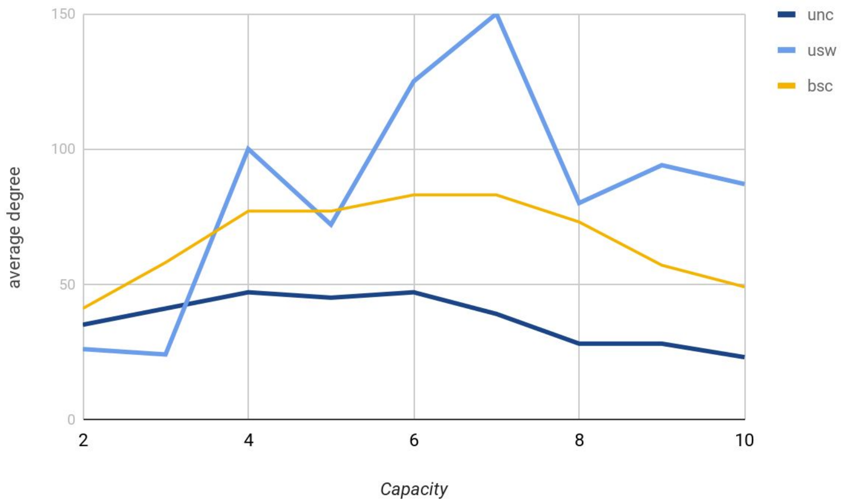




Topological properties of LONs

Mean average degree $\overline{z}$:

- $\overline{z}$ increases with the capacity class
  - Decreases when the capacity class reaches 6

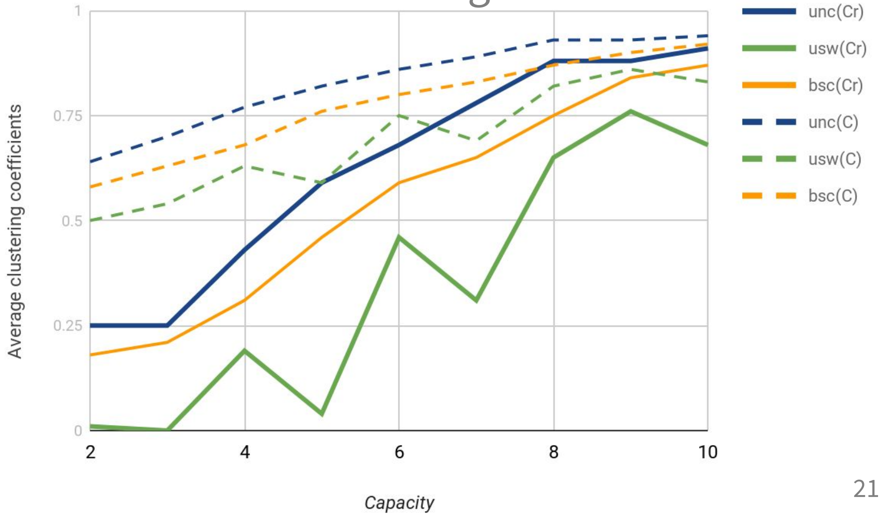


Topological properties of LONs

Mean average clustering coefficients :

- $\overline{C}$: Average clustering coefficients of generated LONs
- $\overline{C_r}$: Average clustering coefficients of corresponding random graphs
    - Random graphs with the same number of vertices and mean degree

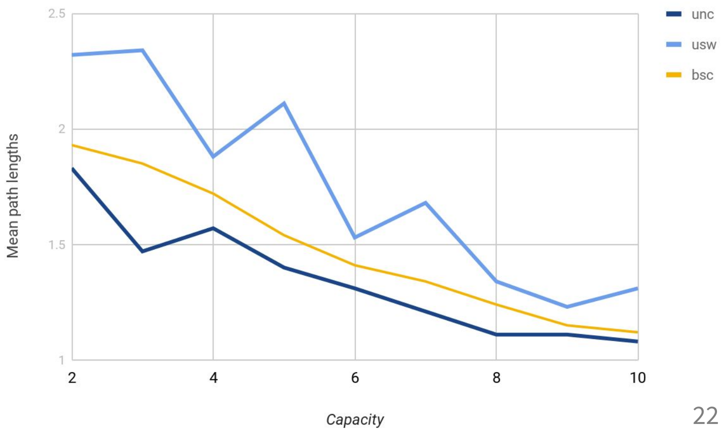
- Local optima are connected in two ways
  Dense local clusters and sparse
  Interconnections
    - Difficult to find and exploit



Topological properties of LONs

Mean path lengths : $\bar{l}$

- All the LONs have a small mean path length
  - Any pair of local optima can be connected by traversing only few other local optima.
- $\bar{l}$ is proportional to log(n_v)
- A sophisticated local search-based metaheuristics with exploration abilities can move from a local optima to another only in few iterations



Topological properties of LONs

Connectivity rate π / number of subgraphs : $\overline{S}$

- The connectivity rate shows that all the LONs generated using J2B are fully connected
- Some of the LONs generated using JIB are disconnected graphs with a significantly high number of non-connected components

Degree Distributions

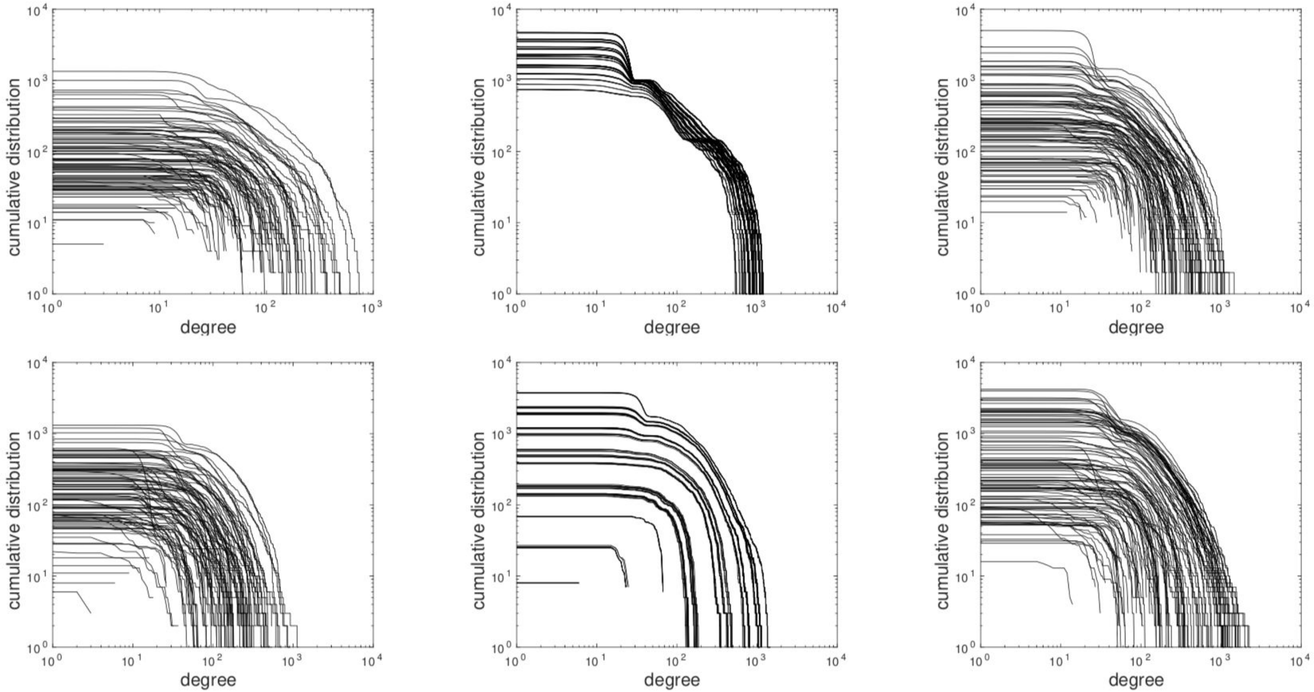


Figure 2: Cumulative degree distribution of J2B (top) and JIB (bottom) for $\mathcal{T}$ = *unc*, C = 5 (left), $\mathcal{T}$ = *usw*, C = 5 (middle), and $\mathcal{T}$ = *bsc*, C = 5 (right). All curves are shown in a log-log scale.

Degree Distributions

Degree distributions decay slowly for small degrees, while their dropping rate is significantly faster for high degrees

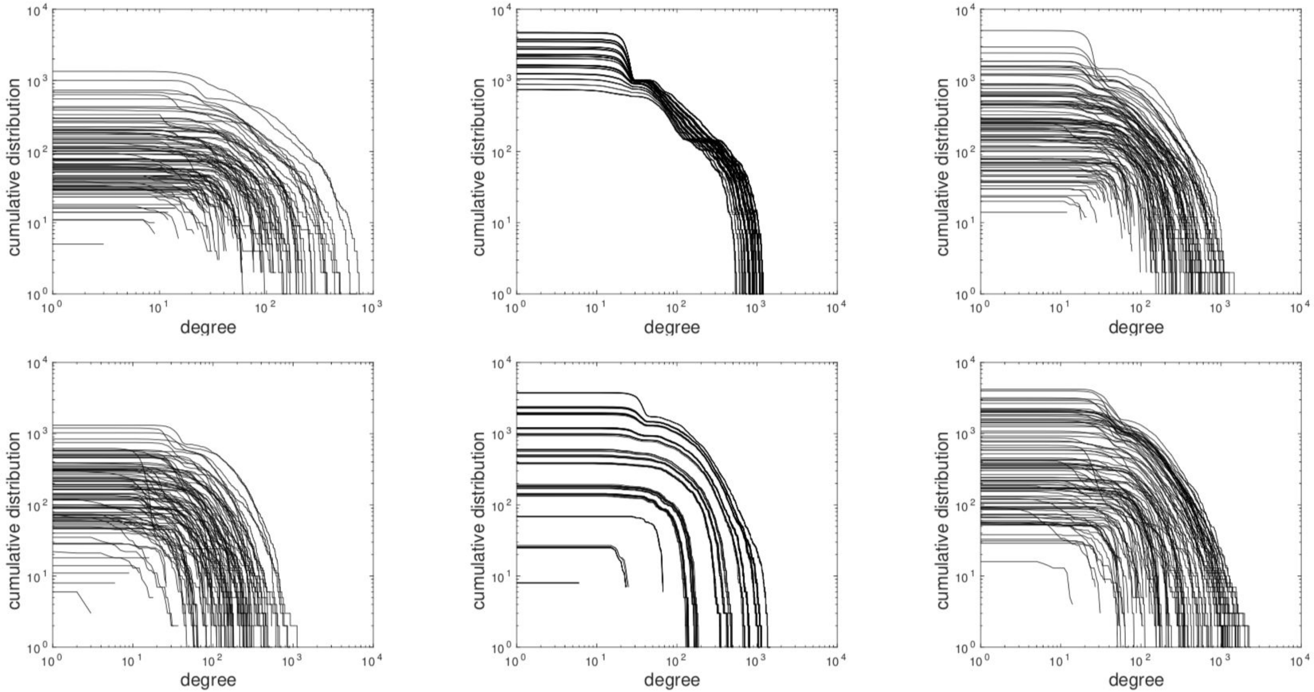


Figure 2: Cumulative degree distribution of J2B (top) and JIB (bottom) for $\mathcal{T} = unc$, $C = 5$ (left), $\mathcal{T} = usw$, $C = 5$ (middle), and $\mathcal{T} = bsc$, $C = 5$ (right). All curves are shown in a log-log scale.

Majority of LO have a small number of connections , while a few have a significantly higher number of connection.

Degree Distributions

Do the distributions fit a power-law as most of the real world networks?

J2B -> A power law cannot be generalised as a plausible model to describe the degree distribution for all the landscape.

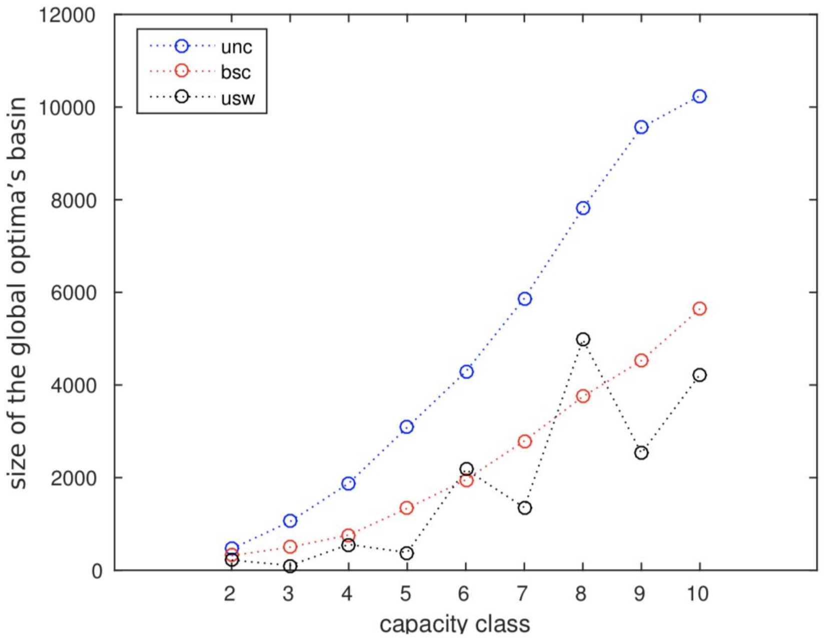
Kolmogorov-Smirnov always fails to reject the exponential distribution as a plausible model for all the samples considered.

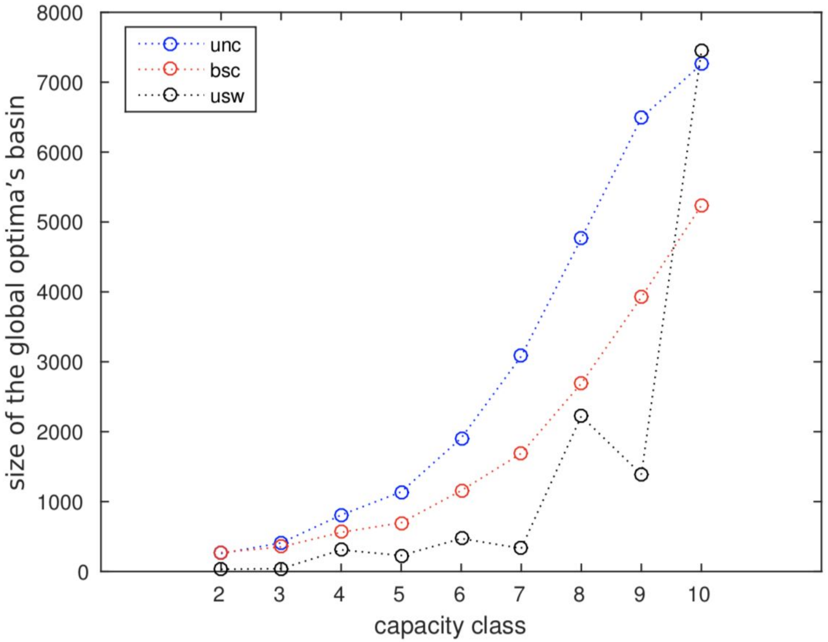
Table 3: The rates at which the Kolmogorov-Smirnov test fails to reject power-law and exponential as plausible distribution models, with a significance level of 0.1

		T=unc, C=5	T=usw, C=5	T=bsc, C=5
Power-law	J2B	0.22	1	0.53
	JIB	0.39	0.26	0.46
Exponential	J2B	1	1	1
	JIB	1	1	1

Basins of attraction

Average of the relative size of the basin corresponding to the global maximum for each capacity C over the 100 TTP instances for J2B (left) and JIB (right).
In all cases: as the capacity C gets larger, the global optima's basins get larger. (search space size per instance: 46080)

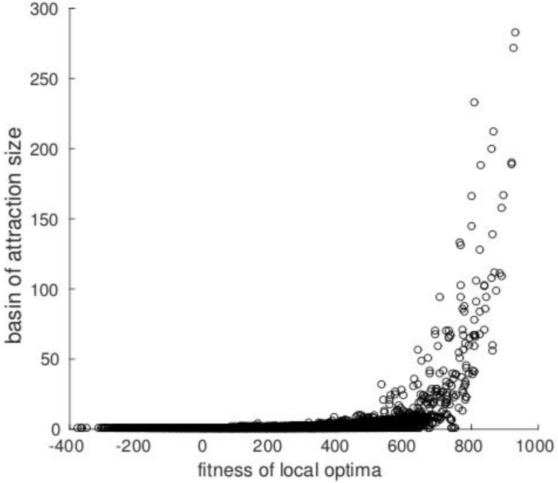


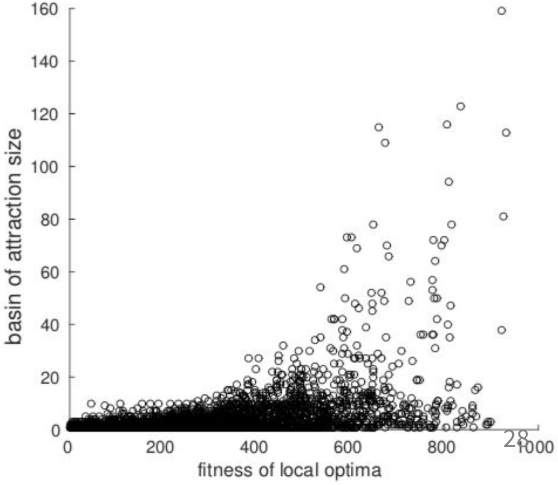


Basins of attraction

Correlation of fitness (x-axis) and basin size (y-axis);
J2B (top) and JIB (bottom).

Good correlation can be exploited: get a rough idea (on-the-fly) about achievable performance, and based on this restart dynamically.
[our conjecture, to be implemented]





Conclusions

Conclusions and Future Directions

- Enumerable TTP instances: local area networks created for two heuristics

- Identified characteristics for hardness:
  - Disconnected components
  - Sometimes low correlation of fitness and basin size -> allows for fitness-based restarts?
  - Easier: large knapsack capacities (larger basins of attraction and overall smaller networks)

- Future work
  - There are (sometimes) many local optima with very small basins -> Tabu search based on tracked paths and distances to local optima?

- Source code: https://bitbucket.org/elkrari/ttp-fla/

Thank you !

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