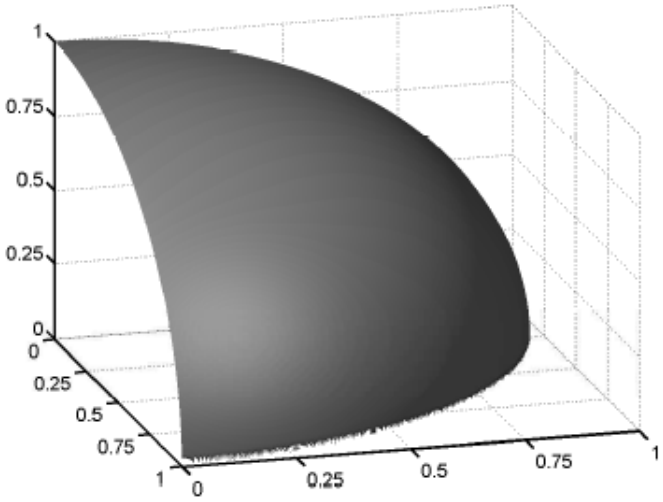
Markus Wagner and Frank Neumann

A Fast Approximation-Guided Evolutionary Multi-Objective Algorithm

THE UNIVERSITY of ADELAIDE

Introduction



- Pareto front: set of all the (many) different trade-offs

- EMOAs restrict themselves to a smaller set that should be a good approximation of the Pareto front

- Different EMOAs (e.g., NSGA-II, SPEA2, IBEA, SMS-EMOA, MOEA/D, ...) try to achieve approximations by preferring diverse sets of non-dominated solutions.

- The typical lack of a formal notion of approximation makes it hard to evaluate and compare algorithms for MOO problems.

Approximation-Guided Evolution (AGE)

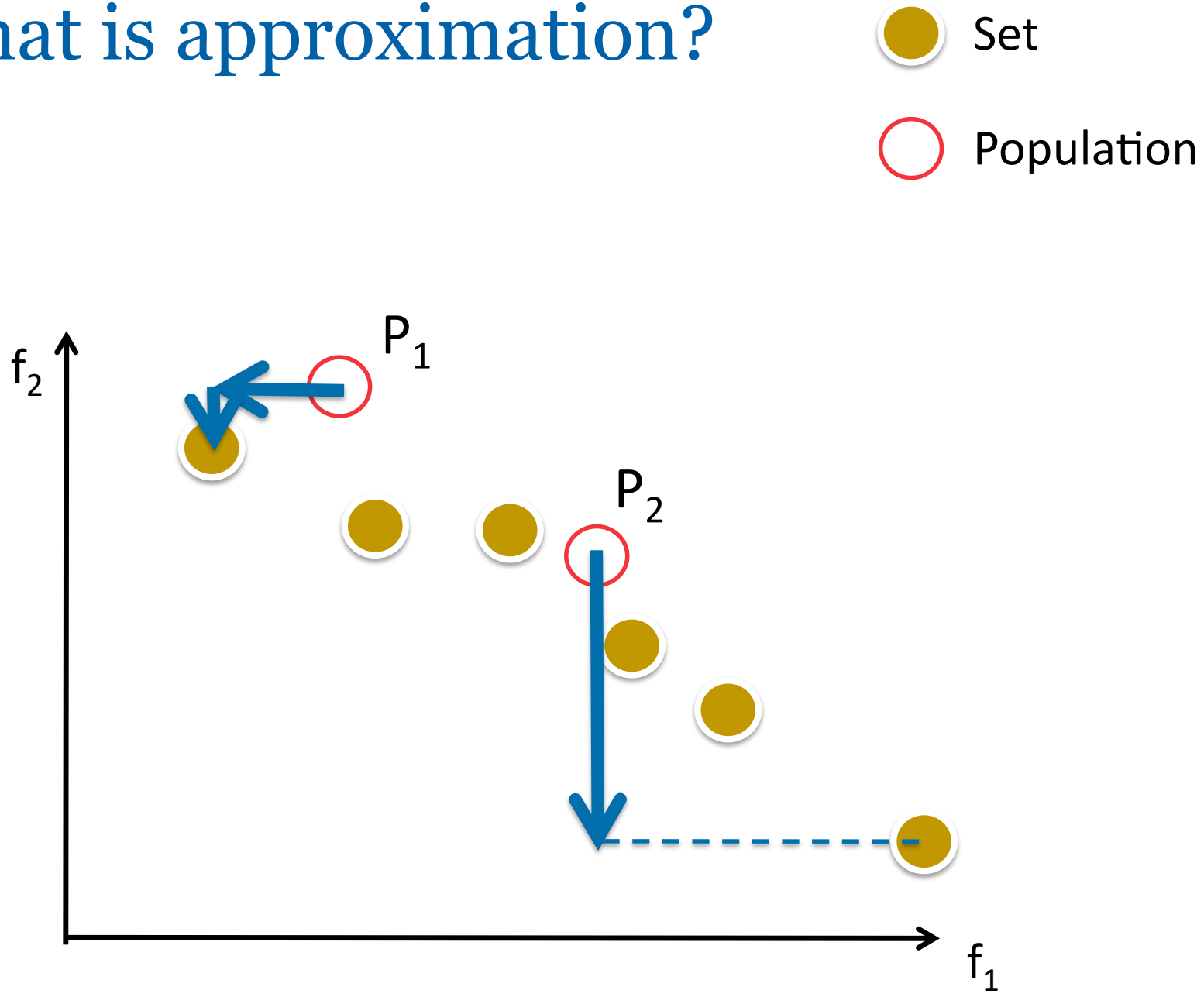
- Motivated by studies in theoretical computer science
  [initially: formal notion
  then: comparison with the hypervolume indicator
  now: an efficient framework]
  [Cheng, Janiak, Kovalyov 1998]
  [Papadimitriou, Yannikakis 2000, 2001]
  [Vasilvitskii, Yannakakis 2005]
  [Diakonikolas, Yannakakis 2009]
  [Daskalakis, Diakonikolas, Yannakakis 2010]
  [Bringmann, Friedrich 2010]
- The AGE framework
  - allows to incorporate a formal notion of approximation
  - improves the approximation quality iteratively
  - uses the best knowledge obtained so far (“archive”)
- Given a fixed time+evaluations budget, AGE outperforms other EMOAs in terms of the desired additive approximation, as well as the covered hypervolume (see our IJCAI ‘11 article)

Contribution

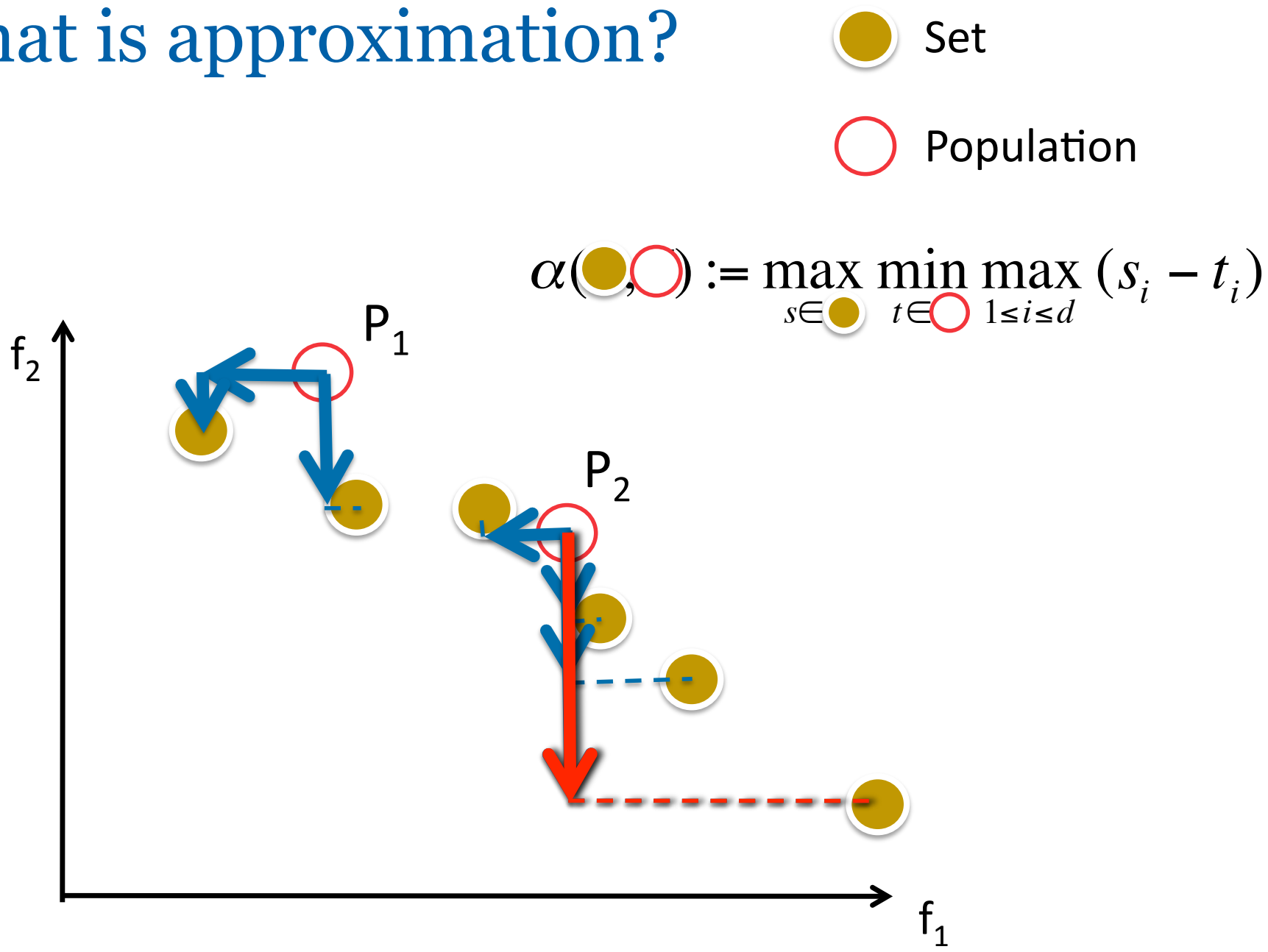
- Approximation-Guided Evolution (AGE) [IJCAI '11]
  1. Its runtime of AGE can suffer in high-dimensional spaces
  2. It has a mediocre performance on "easy" problems

- Approximation-Guided Evolution II (AGE-II) **today***
  1. It limits the archive size through the ε-dominance approach
  2. Introduction of non-random parent selection that is not detrimental in high-dimensional spaces

What is approximation?



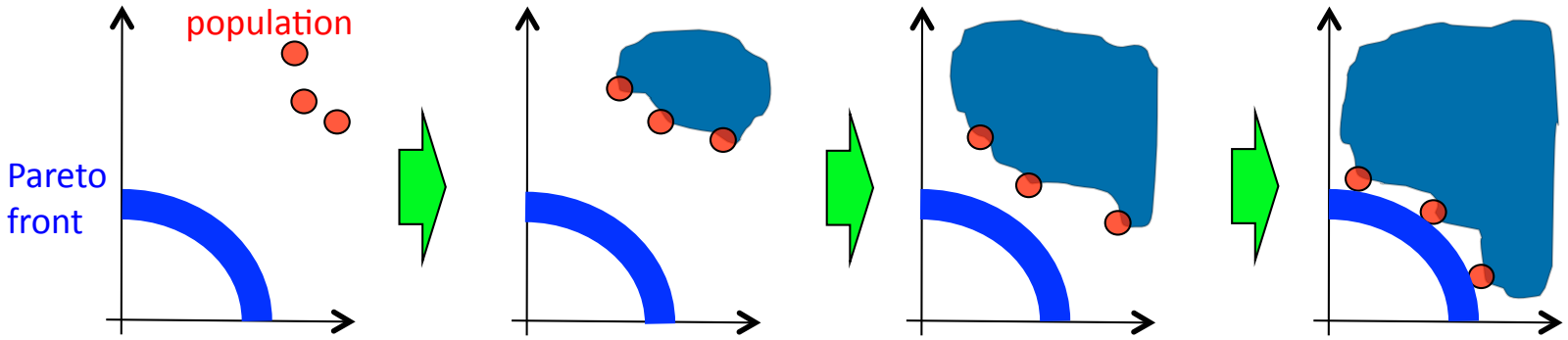
What is approximation?



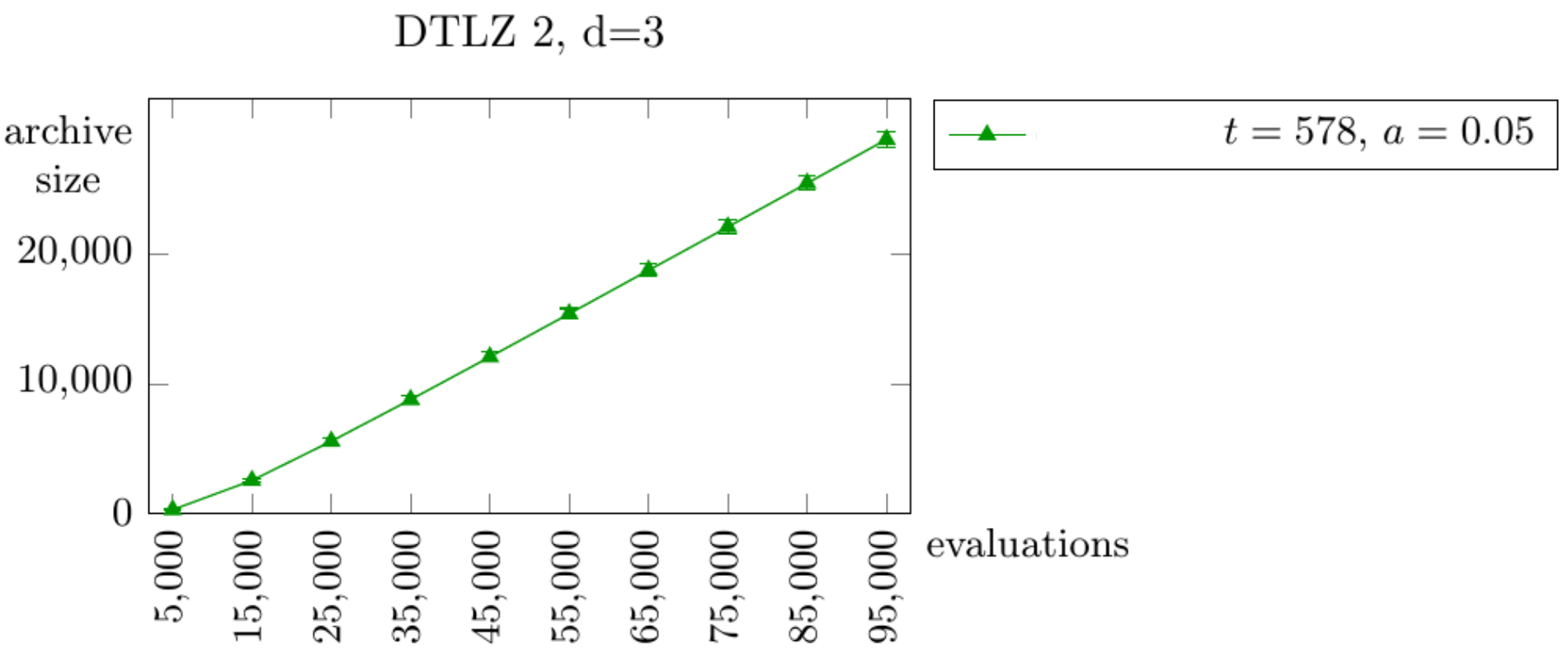
Approximation-Guided EA

α(f,X) is the approximation ratio achieved by the set X with respect to the Pareto front f

- Aim: find X such that α(f,X) is minimised
- Problem: we do not know the Pareto front f
- Solution: use the union of all non-dominated points seen so far ("archive") as an approximation of the Pareto front f

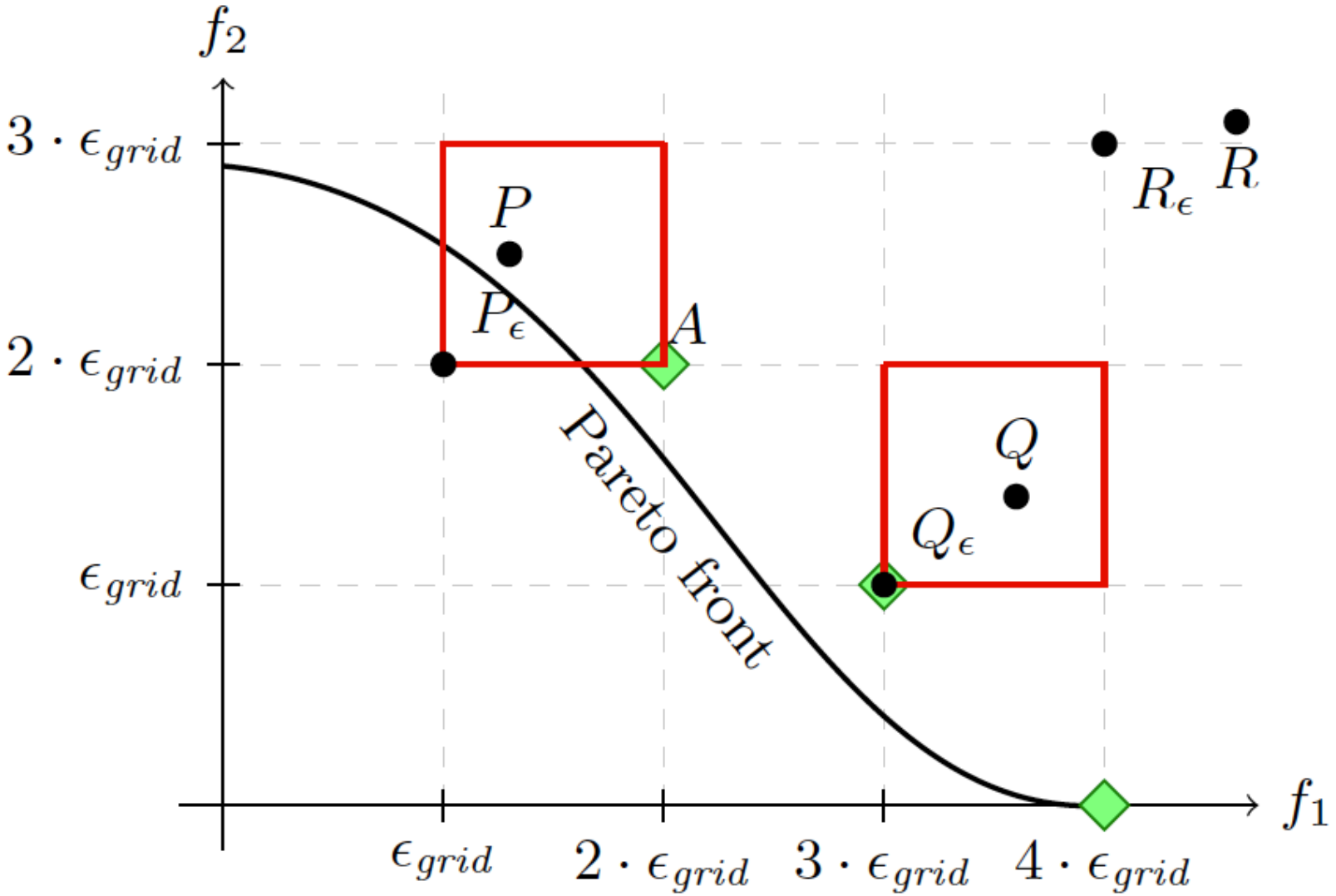


Development of the Archive Size

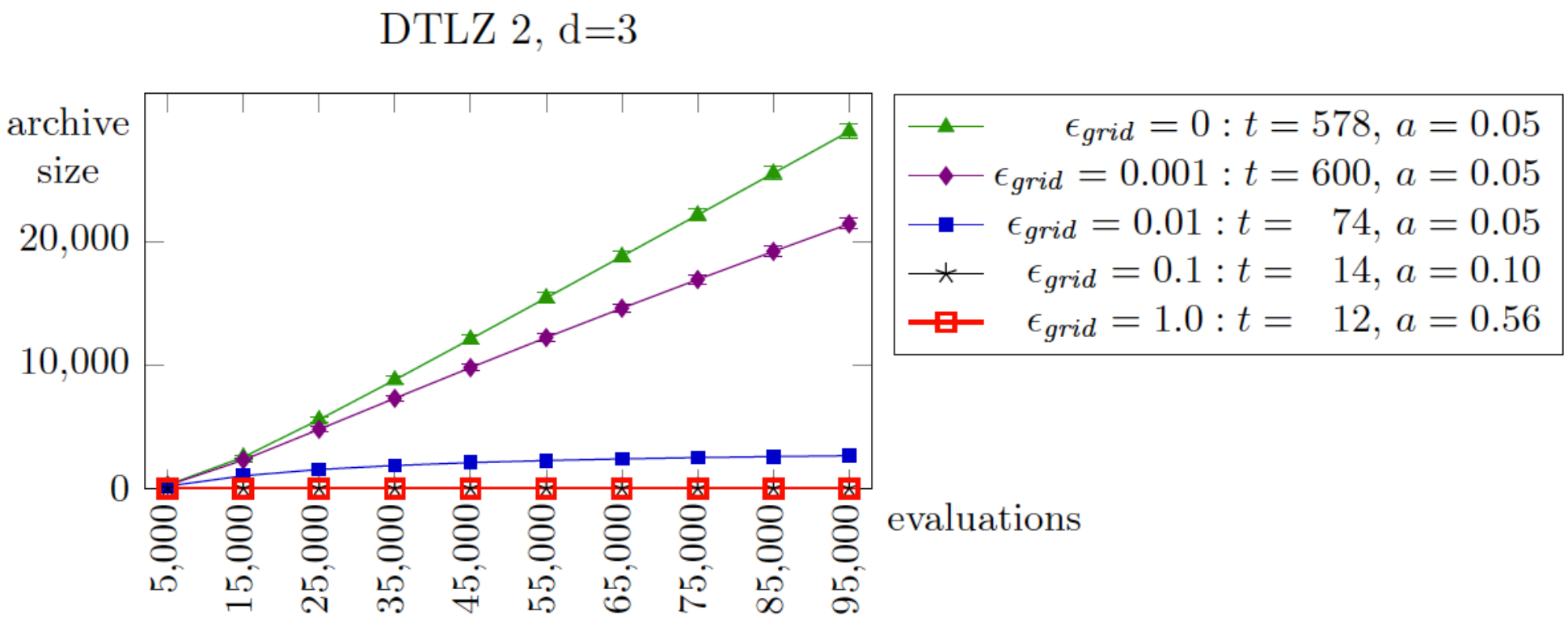


100.000 evaluations, averages of 100 independent runs

ε-Dominance Approach [based on Laumanns et al. '02]

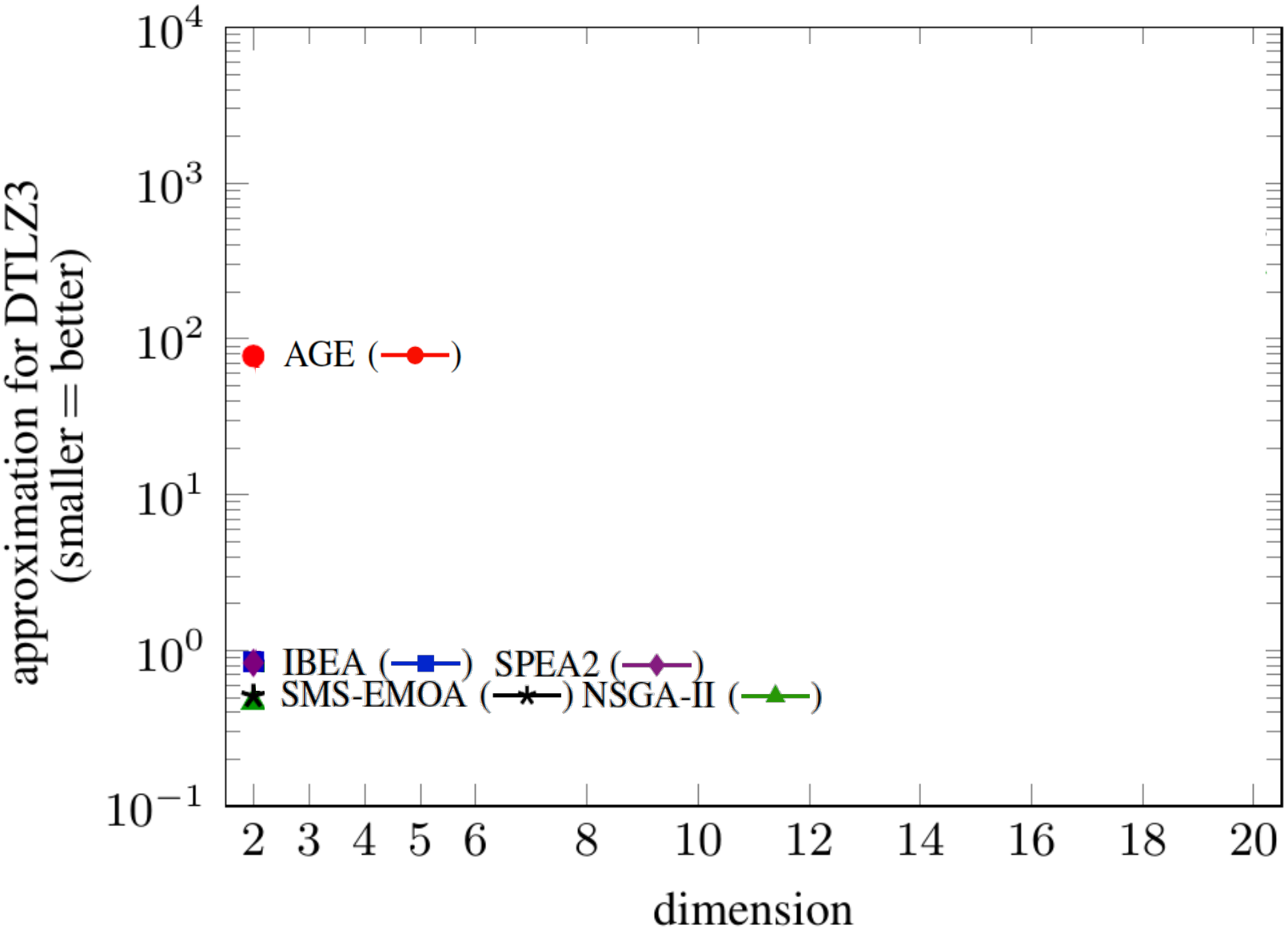


Development of the Archive Size

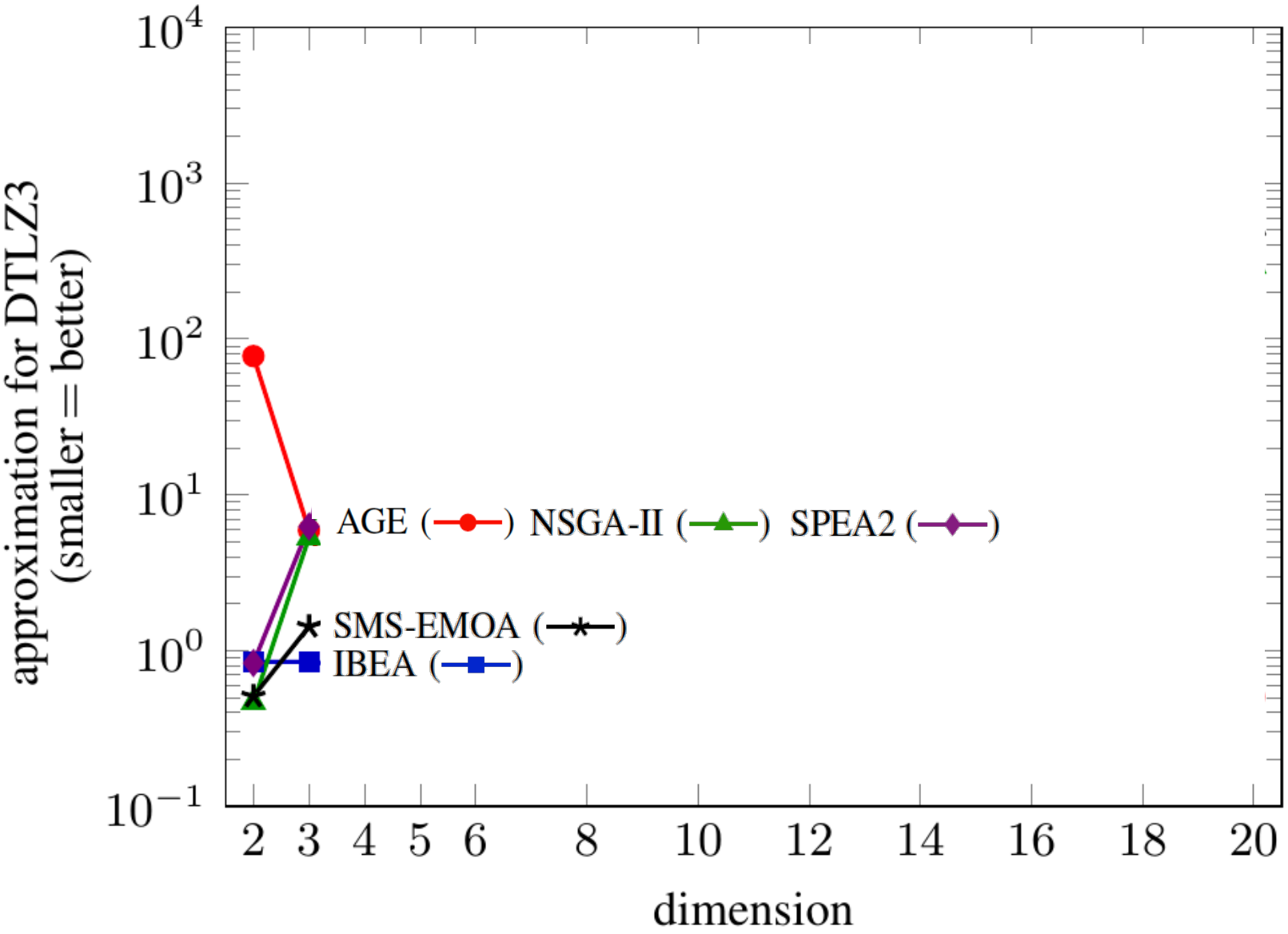


100.000 evaluations, averages of 100 independent runs (many more plots in this GECCO article)

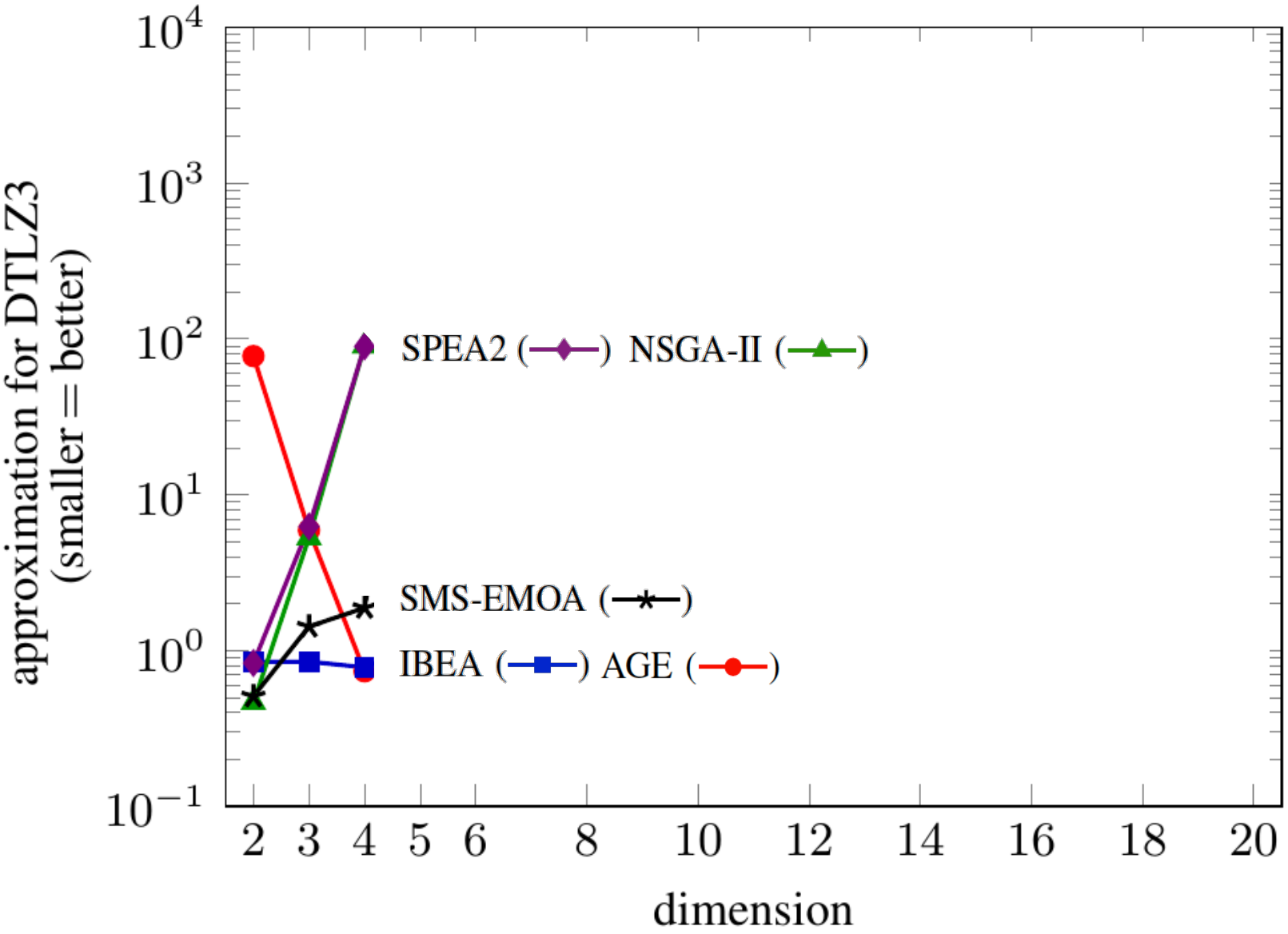
Parent Selection



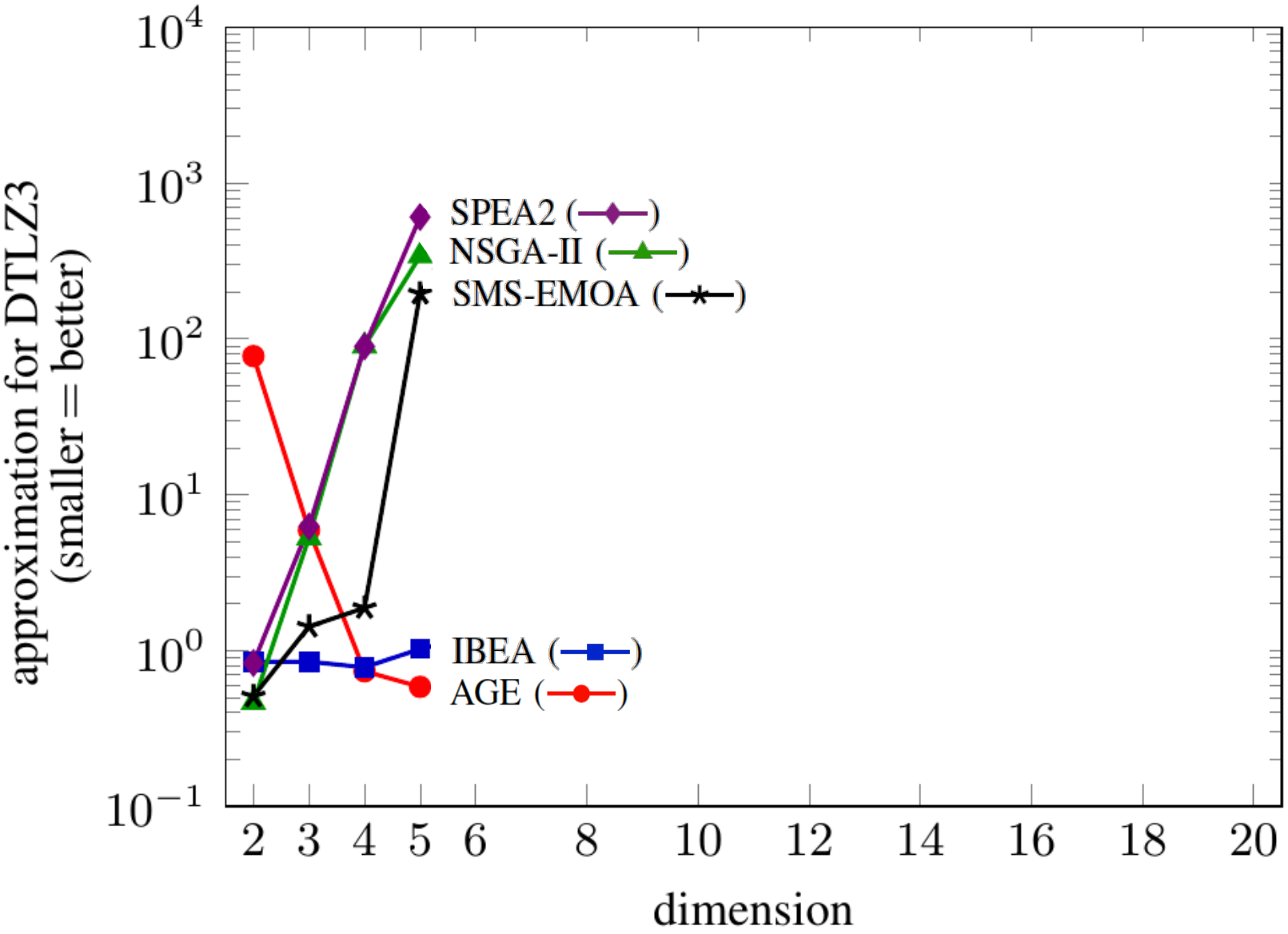
Parent Selection



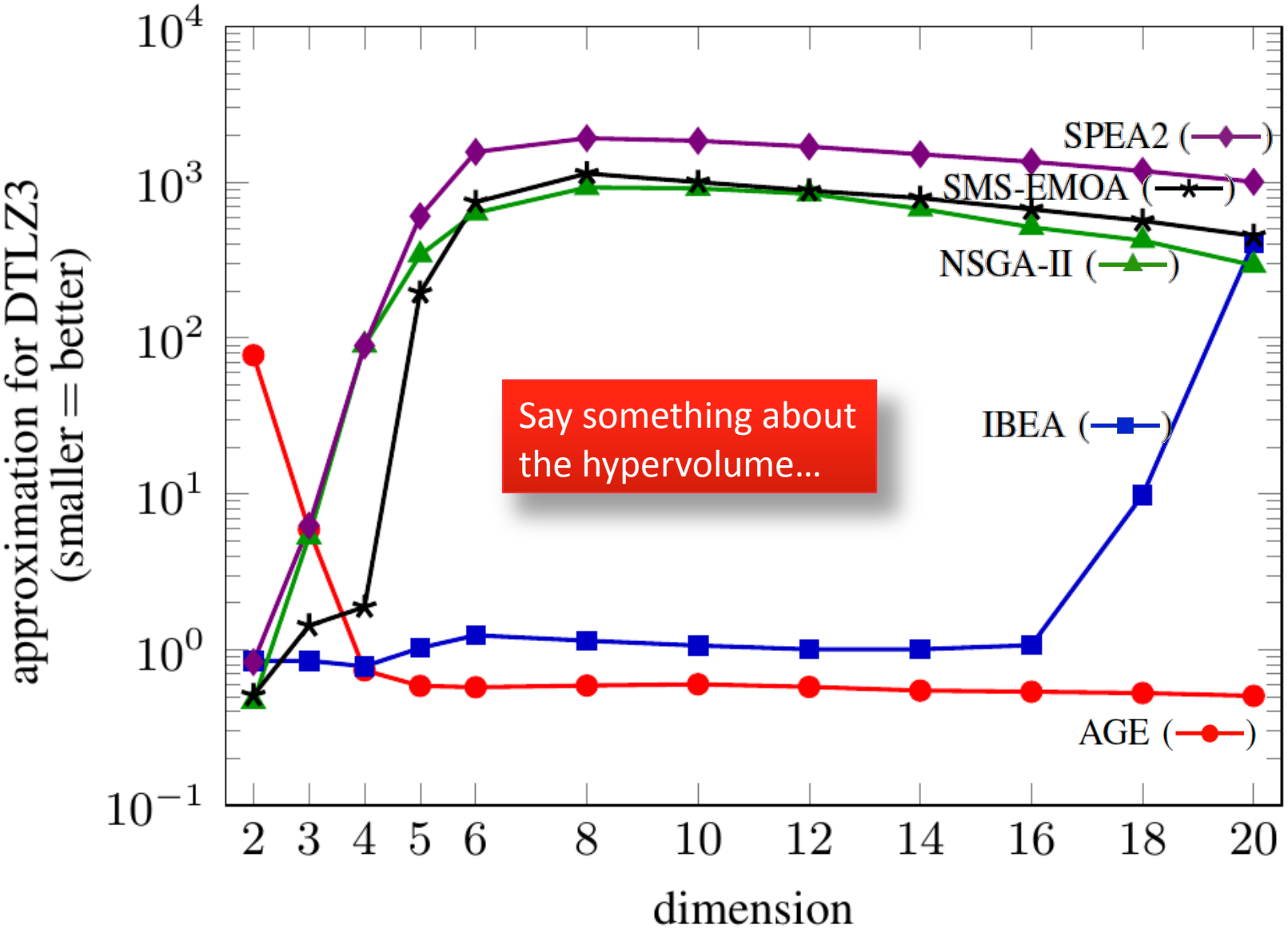
Parent Selection



Parent Selection



Parent Selection



Now
hype
due
of th

Experiments

- NSGA-II, IBEA, SPEA2, SMS-EMOA
  with approx hyp: SMS-EMOA, MO-CMA-ES
  AGE
  AGE-II with $\varepsilon_{grid}=0$, $\varepsilon_{grid}=0.1$, $\varepsilon_{grid}=0.01$

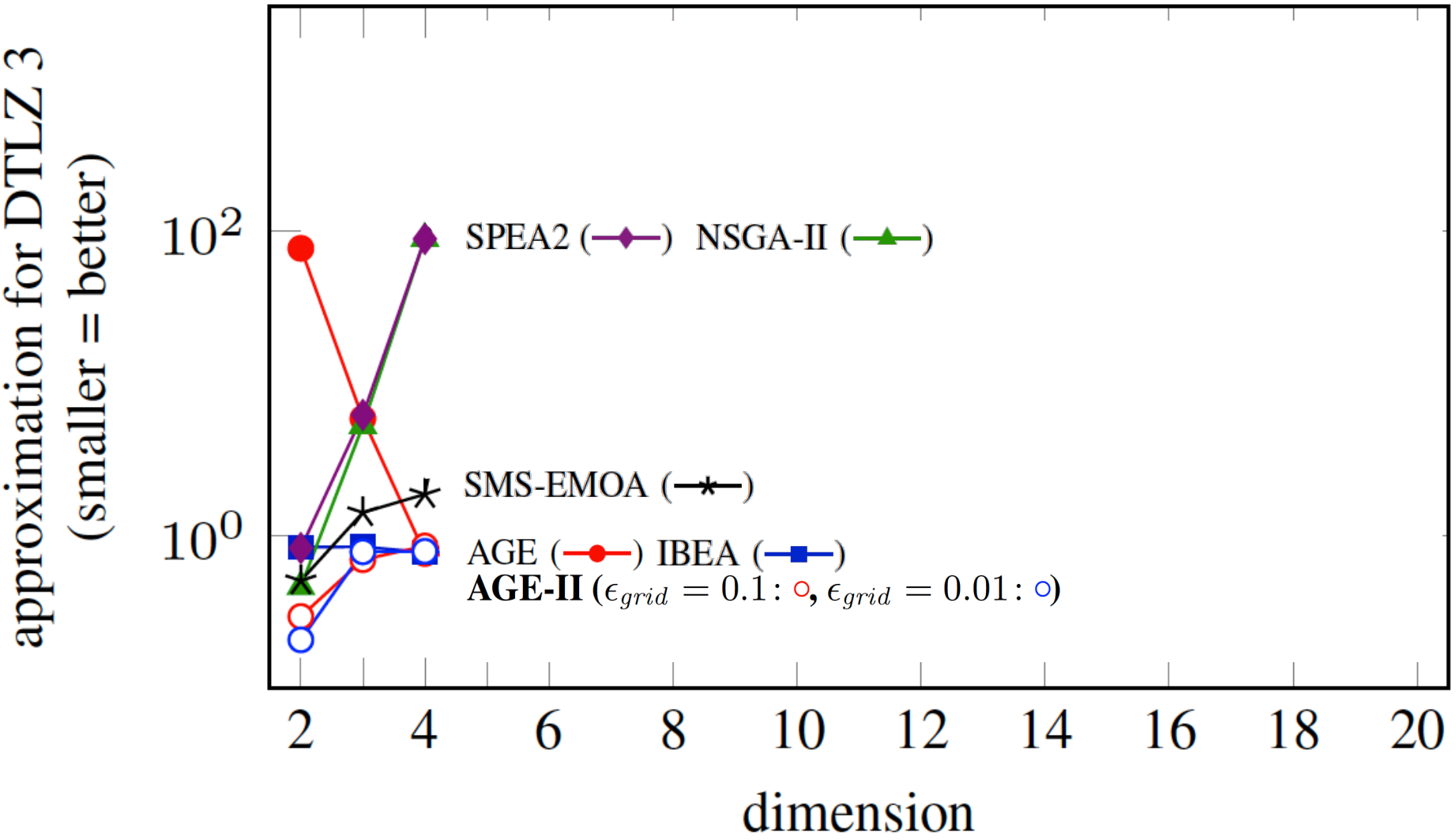
- ZDT 1/2/3/4/6
  WFG 1-9 (each with d=2 and d=3)
  LZ 1-9
  DTLZ 1/2/3/4 (each with d=2,…,20)
  → 80 functions, plenty of plots

- Limits: 4h (and varying numbers of evaluations)

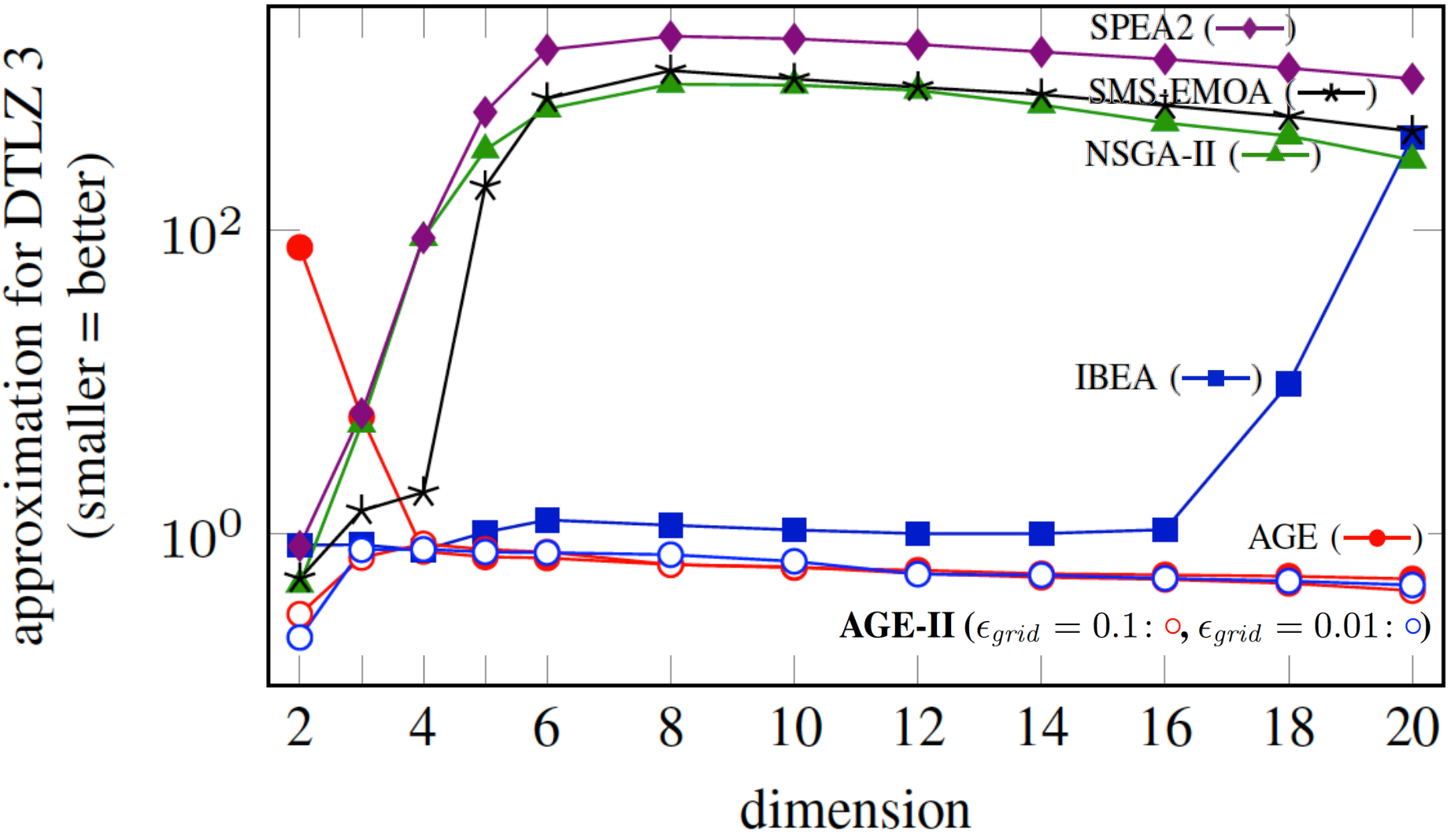
- $\mu=100$, SBX, PM, implemented in jMetal

(Psst… code is available online… **http://tinyurl.com/gecco2013**)

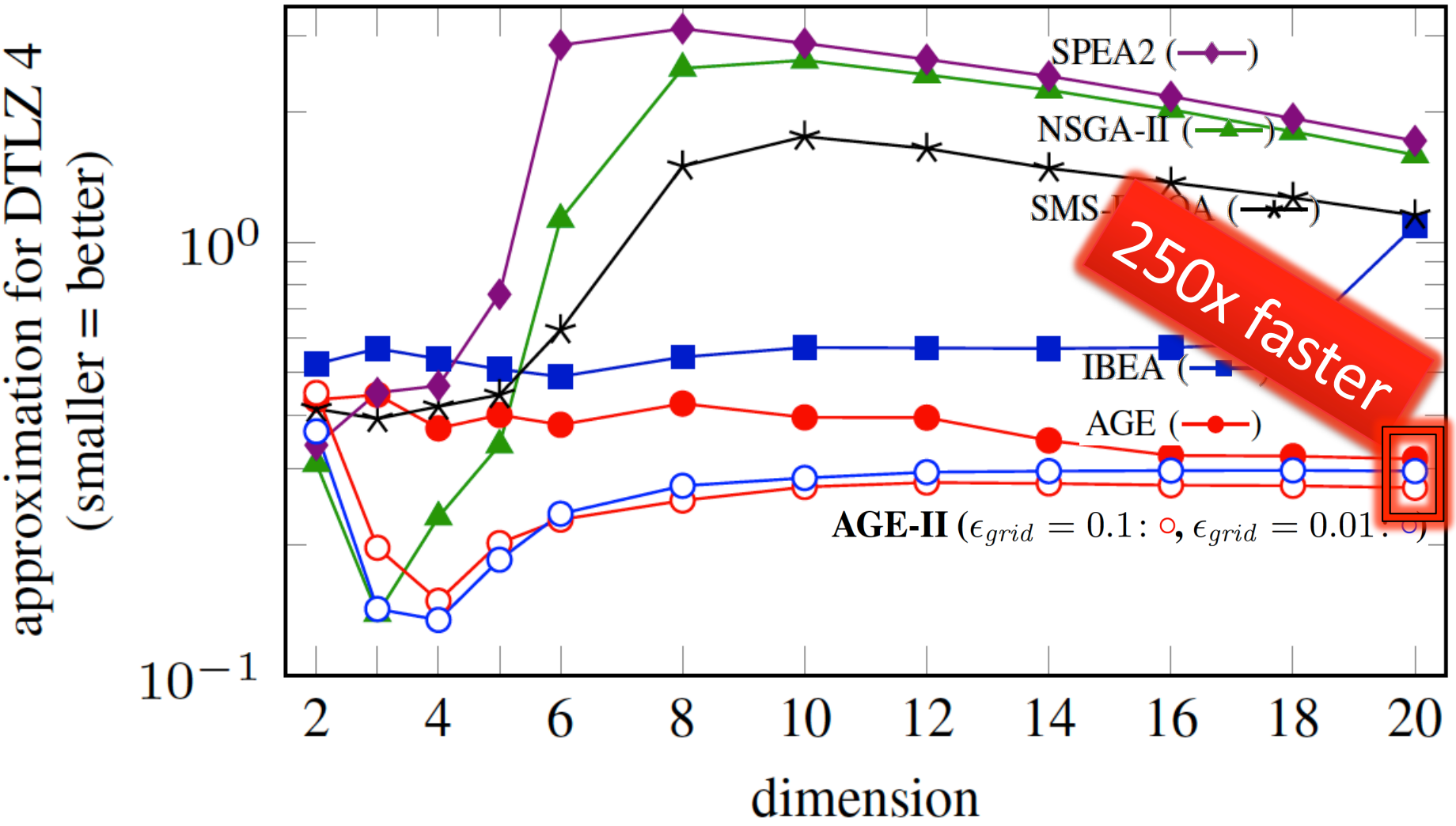
Results


approximation for DTLZ 3
(smaller = better)
10^2
10^0
SPEA2 (—♦—) NSGA-II (—▲—)
SMS-EMOA (—*—)
AGE (—●—) IBEA (—■—)
AGE-II ($\epsilon_{grid} = 0.1$: ○, $\epsilon_{grid} = 0.01$: ○)
2 4 6 8 10 12 14 16 18 20
dimension

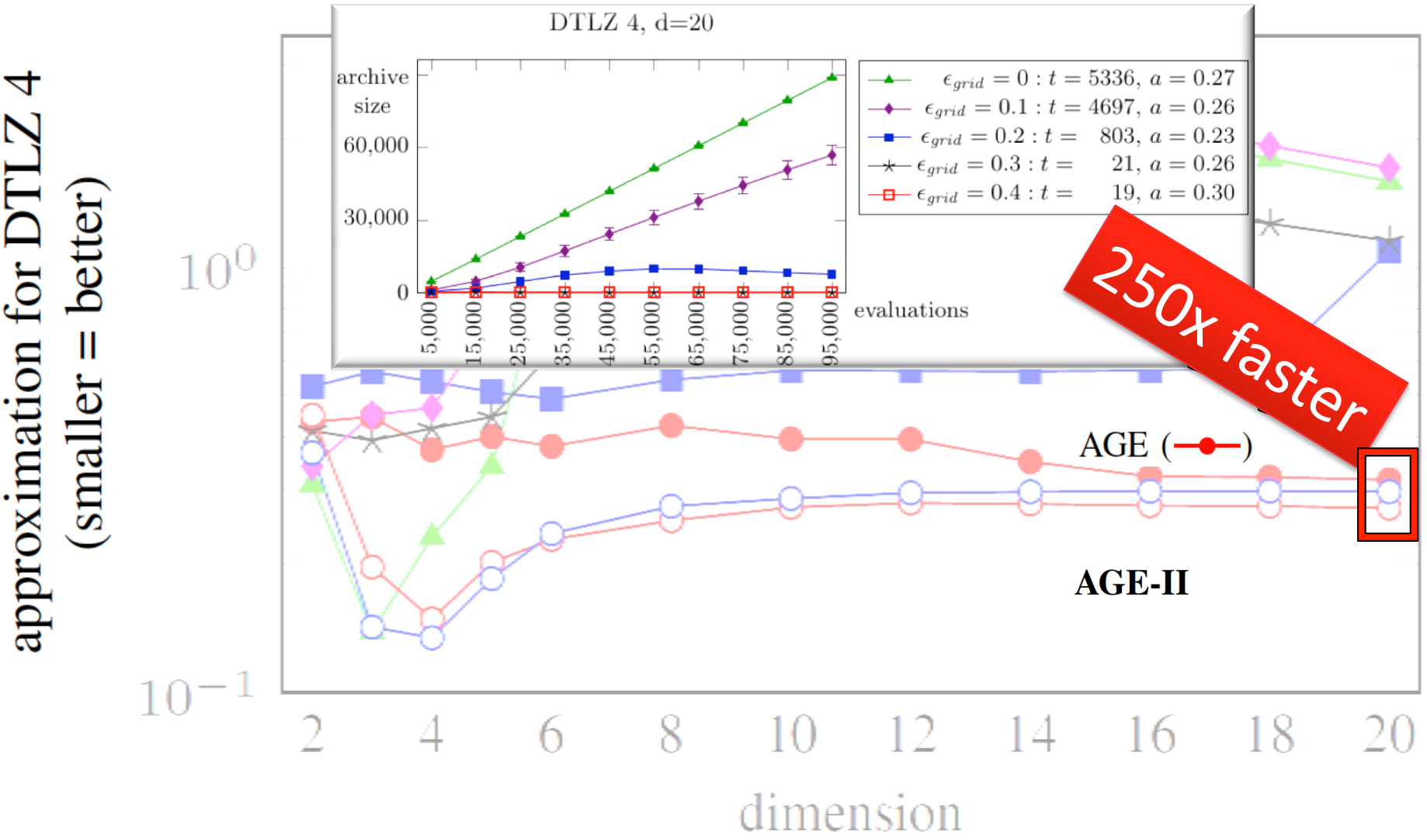
Results


approximation for DTLZ 3
(smaller = better)
10^2
10^0
SPEA2 (—◆—)
SMS-EMOA (—*—)
NSGA-II (—▲—)
IBEA (—■—)
AGE (—●—)
AGE-II ($\epsilon_{grid} = 0.1$: ○, $\epsilon_{grid} = 0.01$: ○)
2
4
6
8
10
12
14
16
18
20
dimension

Results


approximation for DTLZ 4
(smaller = better)
10^0
10^{-1}
2
4
6
8
10
12
14
16
18
20
dimension
SPEA2 (—◆—)
NSGA-II (—▲—)
SMS-
A (—★—)
IBEA (—■—)
AGE (—●—)
AGE-II ($\epsilon_{grid} = 0.1$: ○, $\epsilon_{grid} = 0.01$: ○)
250x faster

Results


approximation for DTLZ 4
(smaller = better)
10^0
10^{-1}
2
4
6
8
10
12
14
16
18
20
dimension
DTLZ 4, d=20
archive size
60,000
30,000
0
5,000
15,000
25,000
35,000
45,000
55,000
65,000
75,000
85,000
95,000
evaluations
$\epsilon_{grid} = 0 : t = 5336, a = 0.27$
$\epsilon_{grid} = 0.1 : t = 4697, a = 0.26$
$\epsilon_{grid} = 0.2 : t = 803, a = 0.23$
$\epsilon_{grid} = 0.3 : t = 21, a = 0.26$
$\epsilon_{grid} = 0.4 : t = 19, a = 0.30$
250x faster
AGE (—●—)
AGE-II

Conclusions

Approximation-Guided Evolution II (AGE-II)

- an efficient approach to solve multi-objective optimisation problems with few and many objectives
- one parameter
- enables practitioners
  1. to **add objectives** with only minor consequences
  2. to **explore** problems for even higher dimensions

Future work

- Use it!
- Code is available online…
  http://tinyurl.com/gecco2013 Java + C
- Bonus: AGE-II will be in the next jMetal version! ☺