A Generic Bet-and-run Strategy for Speeding Up Stochastic Local Search

Markus Wagner

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Code and results: https://bitbucket.org/markuswagner/restarts

Context in this session

Carola: change parameters during a run

Anja: change algorithms during a run

Markus: don’t change anything during a run

Speeding Up Stochastic Local Search

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Restarts

A desktop PC does not work properly
→ we restart it.

Performance of stochastic algorithm and randomized search heuristics unsatisfactory
→ we restart it again and again.

While this approach is well-known, few algorithms directly incorporate such restart strategies.

Potential reason: added complexity of designing an appropriate restart strategy that is advantageous for the considered algorithm.

We are looking for: a generic framework for restart strategies that is not overly dependent on the algorithm used and the problem considered.

https://media.makeameme.org/created/Hello-IT-Have.jpg

Related work

Luby, Sinclair, and Zuckerman (1993)

- for Las Vegas algorithms with known run time distribution: sequence of running times (1,1,2,1,1,2,4,1,1,2,1,1,2,4,8,…) optimal restarting strategy (up to constant factors)

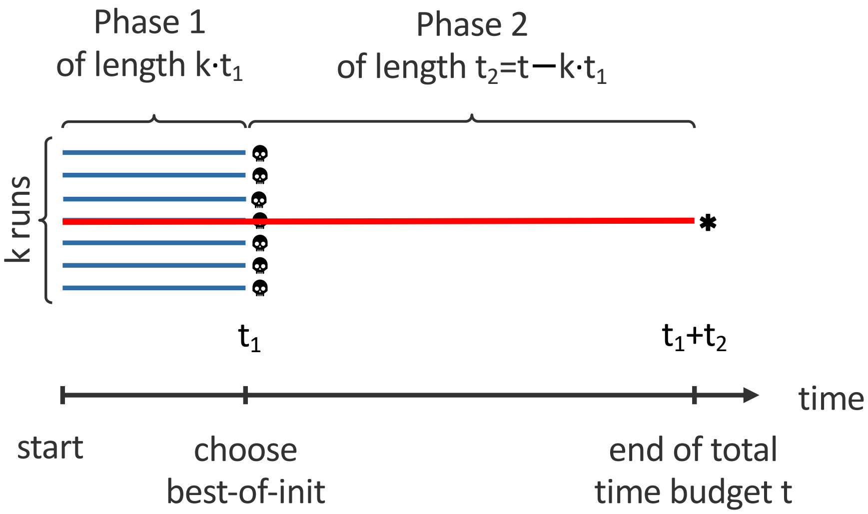
Satisfiability problem

- empirical comparisons showing substantial impact on efficiency of SAT solvers [Biere 2008, Huang 2007]
- unsurprising as SAT/CSP solvers learn no-goods during backtracking [Cireé et al 2014]

Classic optimisation algorithms are often deterministic

- The underlying algorithm of IBM ILOG CPLEX is not random, but characteristics change with memory constraints and parallel computations.
- Lalla-Ruiz and Voss (2016) investigated different mathematical programming formulations to provide different starting points.

Related work
Bet-and-Run by Fischetti and Monaci (2014)



Notes
Single-run:
$k=1$
Multi-run with restarts from scratch:
$t_1=t/k$ and $t_2=0$

Fischetti and Monaci (2014)
"Exploiting erraticism in search"
k=5, CPLEX, diversity, MIPlib 2010

de Perthuis de Laillevault, Doerr, and Doerr (2015)
1+1-EA on OneMax
possible additive runtime gain of order sqrt(n log n)

Related work
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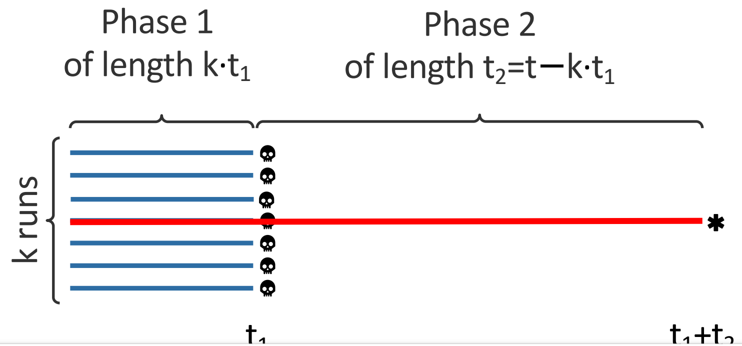

Taking the best of two random samples already decreases the $\Theta(n \log n)$ expected runtime of the (1+1) EA and Randomized Local Search on OneMax by an additive term of order $\sqrt{n}$. The optimal gain that one can achieve with iterated random sampling is an additive term of order $\sqrt{n \log n}$. This also determines the best possible mutation-based EA for OneMax, a question left open in [Sudholt, IEEE TEC 2013].

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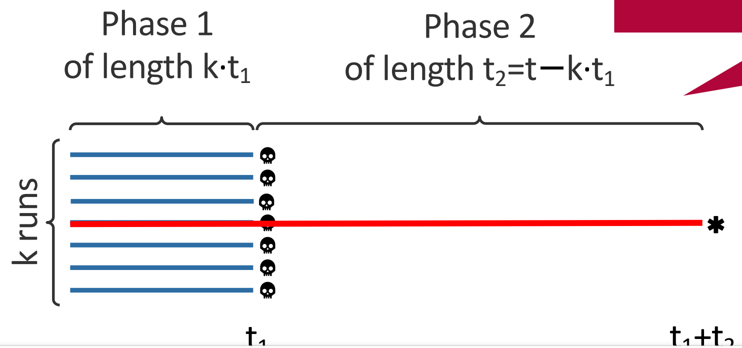
Implementation Detail:

The initial runs can be run sequentially – they don't have to be in parallel. Keep in mind: our goal is to make best use of some total computation budget t, not of some wallclock time.



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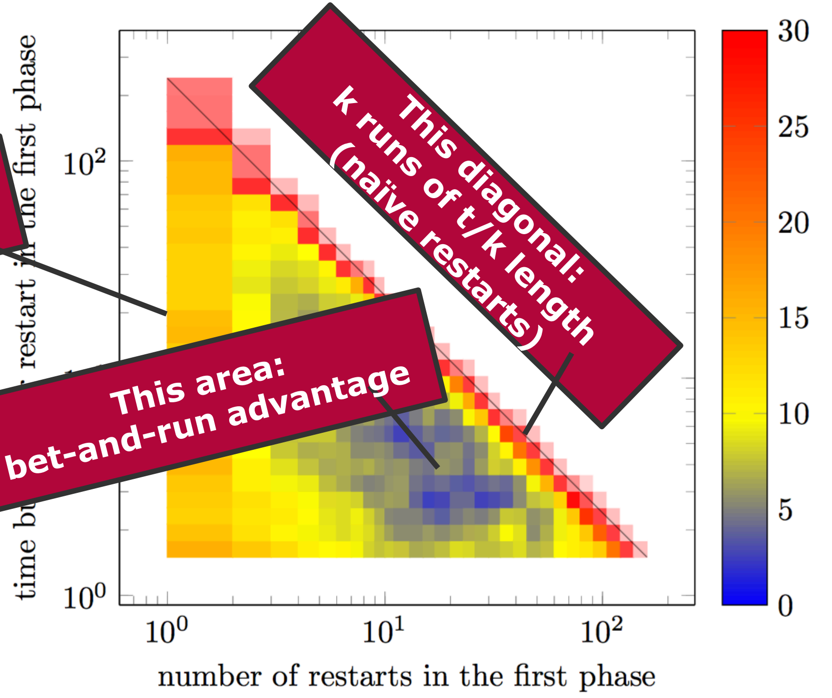
A Generic Bet-and-Run Strategy

Our experiments

- Traveling Salesperson
  - Lin-Kernighan Heuristic (from Concorde)
  - **111** symmetric TSPlib instances with up to 100k cities
- Minimum Vertex Cover
  - FastVC (Cai 2015)
  - **86** large MVC instances

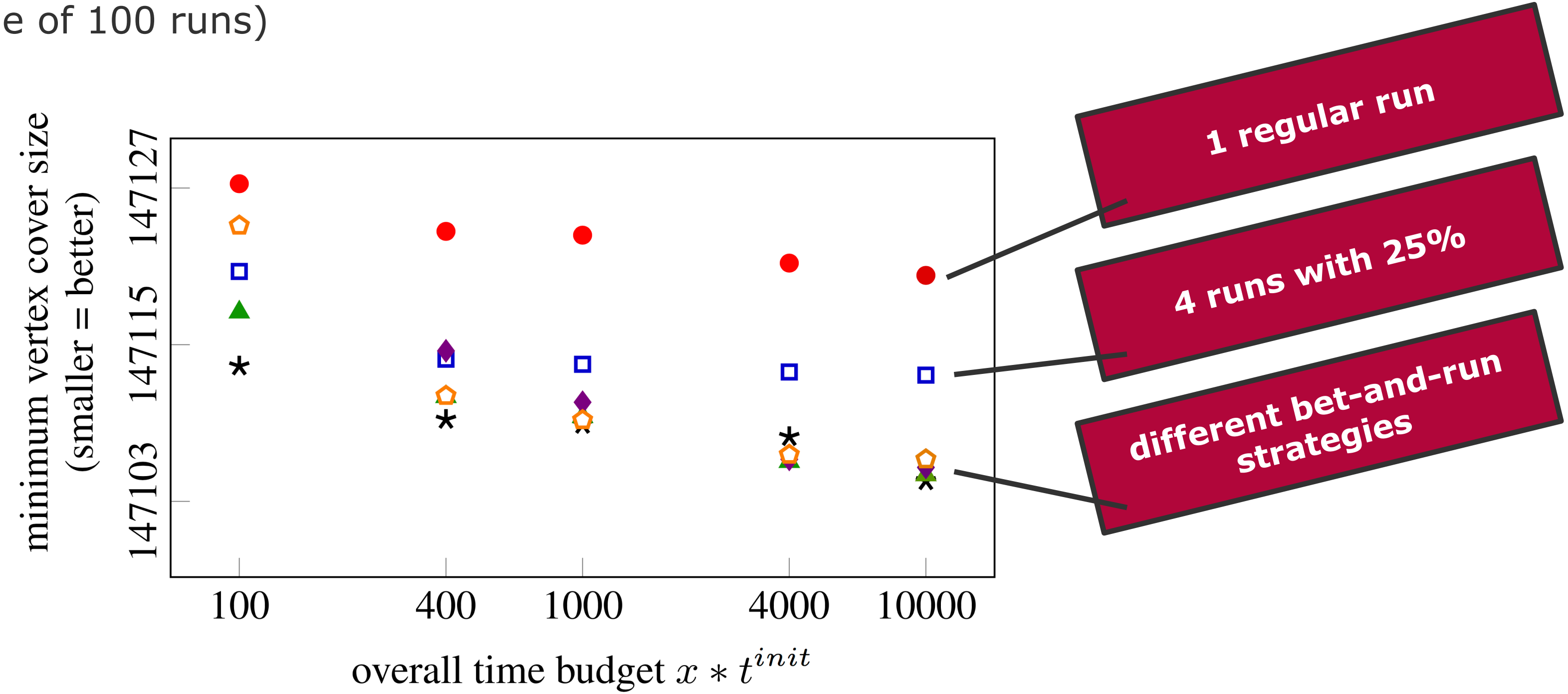
- Also, algorithms are **pure black boxes**:
  start with seed … … … … … stop
- Lots of bet-and-run strategies
  Example: heatmap on the right
  ~450 bet-and-run setups for 1 instance

Example: FastVC on MVC instance shipsec1.mtx
Total budget t=240s
Shown in colour is absolute distance to best-found (117,366).



A Generic Bet-and-Run Strategy
Dependency on total time budget

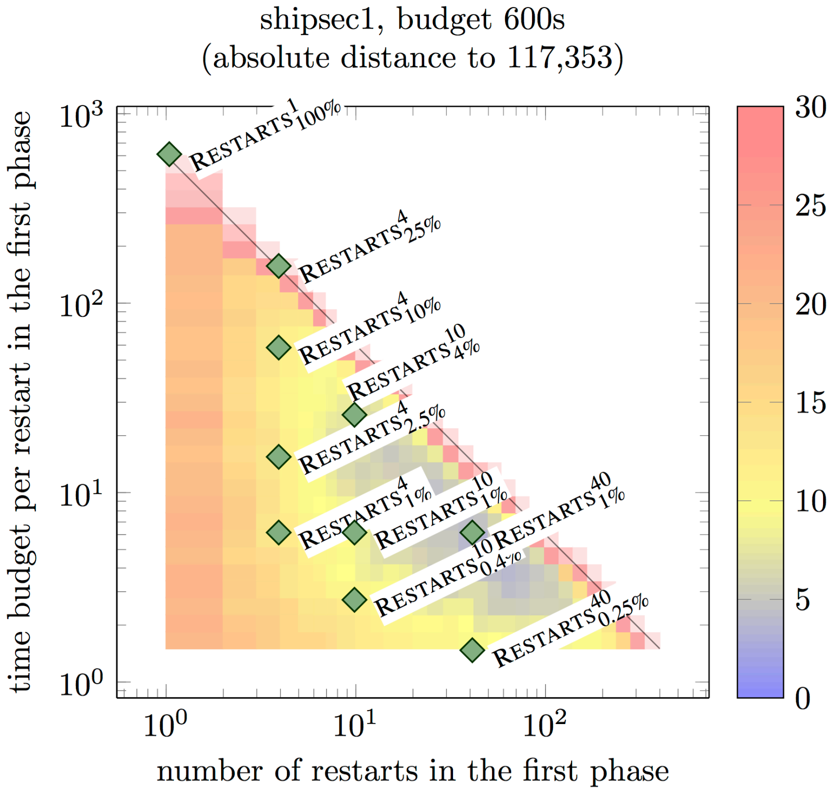
Example: solution quality achieved by FastVC on instance sc-shipsec5 (average of 100 runs)



Cross Domain Study
To-be-investigated Bet-And-Run Approaches

$\textsc{Restarts}^{\mathrm{k}}_{\mathrm{x}\%}$ refers to the strategy where k initial runs are performed, and each of the runs has a computational budget of $x\%$ of the total time budget.

$\textsc{RestartsLuby}^{\mathrm{k}}_{\mathrm{x}\%}$ refers to the strategy that uses in its first phase runs whose lengths are defined by the Luby sequence. k refers to the sequence length used in the first phase, and each Luby time unit is $x\%$ of the total time.



Cross Domain Study
First Results (10 instances per domain only, 14 strategies)

Budget: $400 \cdot t_{\text{init}}$	TSP	MVC
$\text{RESTARTS}^{1}_{100\%}$	12.3	9.4
$\text{RESTARTS}^{4}_{25\%}$	3.0	5.4
$\text{RESTARTS}^{4}_{10\%}$	3.7	4.4
$\text{RESTARTS}^{10}_{4\%}$	2.3	1.8
$\text{RESTARTS}^{40}_{1\%}$	3.0	1.3
$\text{RESTARTS}^{4}_{2.5\%}$	6.2	5.2
$\text{RESTARTS}^{10}_{1\%}$	5.6	3.2
$\text{RESTARTS}^{40}_{0.25\%}$	7.7	3.7
$\text{RESTARTS}^{4}_{1\%}$	9.9	6.5
$\text{RESTARTS}^{10}_{0.4\%}$	9.0	4.5
$\text{RESTARTSLUBY}^{4}_{1\%}$	10.8	6.4
$\text{RESTARTSLUBY}^{10}_{1\%}$	10.3	3.5
$\text{RESTARTSLUBY}^{40}_{1\%}$	7.2	2.5

Shown are average ranks across 10 instances.

More tables in the paper.

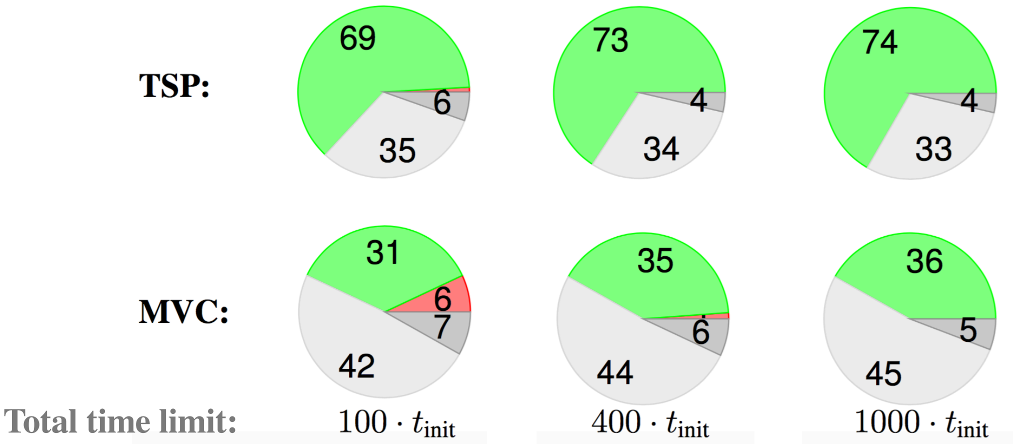
Cross Domain Study
Summary (~200 instances, 1 Bet-and-Run strategy vs 1 single run)

Universally good (given our experiments): Restarts$^{40}{}_{1\%}$

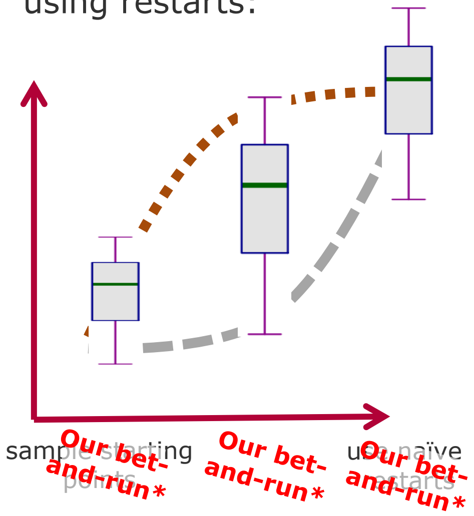
Phase 1: 40 runs, each with a time budget of 1% of the total time budget
Phase 2: use the remaining 60% to continue the best run of Phase 1

Comparison of our "universal" Restarts$^{40}{}_{1\%}$ with a single run:

Wilcoxon-rank-sum test (p=0.05): green shows where Restarts$^{40}{}_{1\%}$ is significantly better,
grey (identical or insignificant), red (single run is better)



Exploitable erraticism using restarts:



Summary so far

We studied a generic bet-and-run restart strategy

- easy to implement as an additional speed-up heuristic
- demonstrated effectiveness on two classical NP-complete optimisation problems with state-of-the-art solvers
- Significant advantage of **Restarts$^{40}_{1\%}$:**
  Phase 1: 40 runs with 1% each of the total time
  Phase 2: continue the best of these 40 for 60% of the total time

Published:

AAAI Conference on Artificial Intelligence 2017
A Generic Bet-and-run Strategy for Speeding Up Stochastic Local Search
Tobias Friedrich, Timo Kötzing, and Markus Wagner

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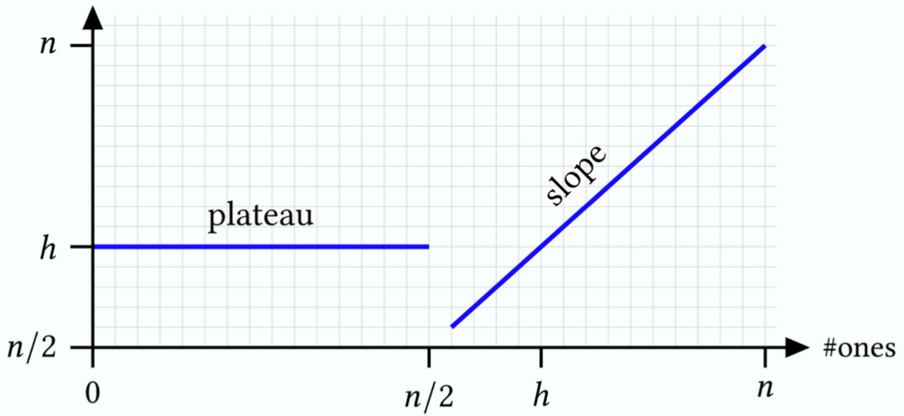
More work on this (1/3) – Theory

Genetic and Evolutionary Computation Conference (GECCO) 2017
Theoretical results on bet-and-run as an initialisation strategy
Andrei Lissovoi, Dirk Sudholt, Markus Wagner, and Christine Zarges



$$f_h(x) = \begin{cases} |x|_1 & \text{if } |x|_1 > n/2 \\ h & \text{otherwise} \end{cases}$$

We define a family of pseudo-Boolean functions (🡐):

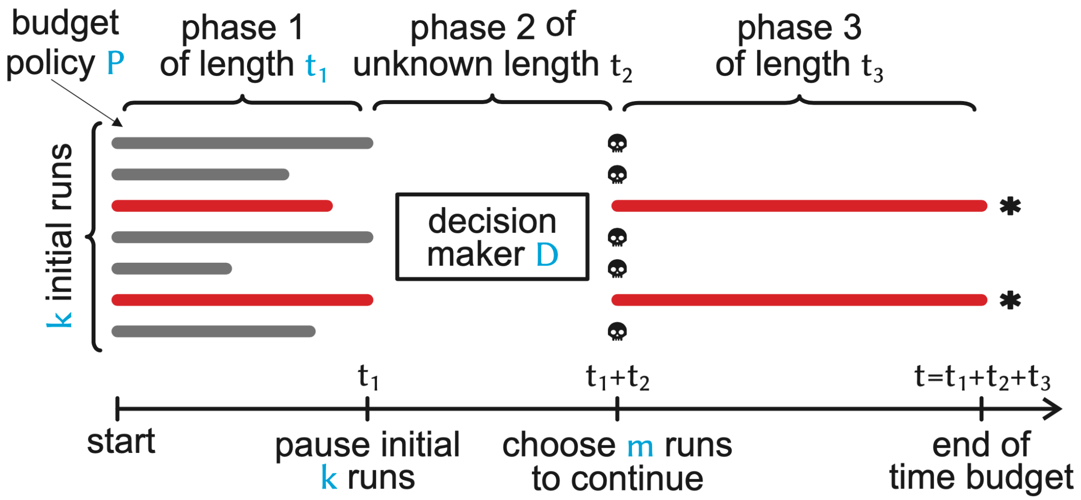
- the plateau shows a high fitness, but does not allow for further progression
- the slope has a low fitness initially, but does lead to the global optimum.

Results:

- non-trivial k and t1 are necessary,
- t1 is linked to properties of the function,
- fixed budget analysis to guide selection of the bet-and-run parameters to maximise expected fitness after t = k · t1 + t2 fitness evaluations.

More work on this (2/3) – Generalised Bet-and-Run

AAAI 2019
An Improved Generic Bet-and-Run Strategy for Speeding Up Stochastic Local Search
Thomas Weise, Zijun Wu, and Markus Wagner



Major result of 78 million experiments:

Decision maker "take current best" is difficult to beat, but it is possible.

More work on this (3/3) – Reactive Restarts

Learning and Intelligent Optimisation (LION) 2017
Learning a Reactive Restart Strategy to Improve Stochastic Search
Serdar Kadioglu, Meinolf Sellmann and Markus Wagner

Drawback of previous work: Whether a run looks promising or abysmal, it gets run exactly until the predetermined limit is reached.

We train (offline) a controller. It then decides online:

1. Continue the current run.
2. Continue an old run.
3. Start a new run.

→ It considers: performance and performance projections of the individual runs, and the remaining time budget.

More work on this (4/3) – Future work

- Other domains: continuous optimisation, multi-objective optimisation, …
- Heterogeneous setups:
  - different hierarchies/races/… of the independent runs
  - different algorithms
  - different algorithm configurations
  - configure on-the-fly
    ➔ this might be a hot topic, and it has a connection to algorithm control, hyper-heuristics, partial restarts (perturbations), …

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