A Petri Net View of Mobility

Mobile systems

- Expose the interplay between *locality* and *connectivity* (Milner)

- **Connectivity** involves having a reference and being able to dereference it

- **Locality** constrains what you can deference

- A simple and general Petri Net solution has proved elusive
Nets-within-nets paradigm (Hamburg)

- (At least) two levels of nets:
  - **System net** has tokens which are black tokens or object nets
  - **Object nets** have black tokens

- **Reference semantics** – tokens are references to Object nets
- **Value semantics** – tokens are instances of Object nets
- **History process semantics** – tokens are Object net processes

Fig. 1. A Mobile Agent as a Net-Token
Nets-within-nets – reference semantics

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Nets-within-nets – reference semantics

Nets-within-nets – value semantics

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Nets-within-nets – value semantics

- Either value or reference or … semantics
  - Value semantics gives notion of locality
  - Reference semantics gives notion of connectivity
- Limited interaction
  - Object net can only interact with transitions adjacent to place
- Formal results are for very limited examples
  - One system net and one (instance of an) object net
  - Value semantics is more powerful than reference semantics
- Examples with Renew are not very persuasive
Proposal for mobile nets

- Start with *modular nets*
  - have a number of Petri Nets – called *modules* or *subnets*
  - combined by place and transition fusion

- Extend the distinction between a *net* and a *system* …
  - *Subnet* captures the structure of a module
  - *Location* = subnet + fusion context
  - *Subsystem* = location with a non-empty marking

Mail agent – a subnet
Mail system

Subnets
Locations
Subsystems

Fusions
Shifting locations

Nets and locations

- Nets (and subnets) are standard

**Definition 2 (Petri Net)**. A Petri Net (PN) is a tuple $PN = (P, T, W)$ where:

1. $P$ is a finite set of places.
2. $T$ is a finite set of transitions with $P \cap T = \emptyset$.
3. $W : (P \times T) \cup (T \times P) \rightarrow \mathbb{N}$ is an arc weight function.

- Locations can be nested (and have a fusion context)

**Definition 5 (PN Location)**. A Petri Net Location is a tuple $L = (S_L, P_L, T_L, W_L)$ where:

1. $S_L$ is a finite set of locations. We define $loc(L) = \bigcup_{s \in S_L} \text{loc}(s) \cup \{L\}$. We require $\forall s \in S_L : \text{loc}(s) \cap \{L\} = \emptyset$.
2. $(P_L, T_L, W_L)$ is a Petri Net. We define $\text{plc}(L) = \bigcup_{s \in S_L} \text{plc}(s) \cup \{P_L\}$ and $\text{trn}(L) = \bigcup_{s \in S_L} \text{trn}(s) \cup \{T_L\}$.
Mobile systems

- Convenient to specify fusion at the level of the system
  - for convenience we assume transitive closure of place fusion sets
  - require consistency of transition fusion sets

**Definition 6 (Mobile System).** A Mobile System is a tuple $MS = (L_0, PF, TF, M_0)$ where:

1. $L_0$ is a location, called the root location. We define $P = plc(L_0)$ and $T = trn(L_0)$.
2. $PF$ is a set of place fusion sets where $\bigcup_{p \in PF} p = P$ and $\forall p_1, p_2 \in PF : p_1 \cap p_2 \neq \emptyset \Rightarrow p_1 = p_2$.
3. $TF$ is a set of transition fusion sets where $\bigcup_{t \in TF} t = T$ and $\forall t_1, t_2 \in TF : t_1 \cap t_2 \neq \emptyset \Rightarrow |t_1| = |t_2|$.
4. $M_0$ is the initial marking of the location.

Classify places and transitions

- **Local** vs **exported** – determined by size of fusion sets
- **Vacate** vs **Occupy** vs **Regular** – determined by arcs

**Definition 9.** For a Mobile System $MS$ we classify places and transitions as follows:

1. $LP = \{p \in P | \exists p \in PF : pf = \{p\}\}$ is the set of local places.
2. $EP = P - LP$ is the set of exported places.
3. $LT = \{t \in T | \exists t \in TF : tf = \{t\}\}$ is the set of local transitions.
4. $ET = T - LT$ is the set of exported transitions.
5. $VT = \{t \in T | \exists p \in LP : W(p, t) > 0 \land \forall p \in P : C(p, t) = 0\}$ is the set of vacate transitions.
6. $OT = \{t \in T | \exists p \in LP : W(t, p) > 0 \land \forall p \in P : C(p, t) = 0\}$ is the set of occupy transitions.
7. $RT = \{t \in T | \exists p_1, p_2 \in LP : W(t, p_1) > 0 \land W(p_2, t) > 0\}$ is the set of regular transitions.
Well-formed mobile system

- Classification of transitions as *vacate, occupy, regular* is consistent and covers all transitions

**Definition 10 (Well-formed).** A Mobile System MS is well-formed if:

1. All transitions are vacate, occupy or regular transitions, i.e. \( T = VT \cup OT \cup RT \).
2. Vacate transitions empty a location for all reachable markings, i.e. \( \forall L \in \text{loc}(L_0) : \forall t \in VT \cap T_L : \forall M \in [M_0] : M(t)M' \Rightarrow \forall p \in LP \cap plc(L) : M'(p) = \emptyset. \)
3. Occupy transitions fill a location for all reachable markings, i.e. \( \forall L \in \text{loc}(L_0) : \forall t \in OT \cap T_L : \forall M \in [M_0] : M(t)M' \Rightarrow \forall p \in LP \cap plc(L) : M(p) = \emptyset. \)

Isolated subsystem

- An *isolated subsystem* has no effect (directly or indirectly) on the root location
  - it can be ignored for the purposes of reachability analysis

**Definition 11 (Isolated subsystem).** Given a Mobile System MS in marking \( M \), a transition sequence \( t_1t_2...t_n \) is a causal sequence if there are markings \( M_1, M_2, ...M_n \) such that \( M[t_1]M_1[t_2]M_2...[t_n]M_n \), and \( \forall k \in 1...(n - 1) : \exists p \in P : W(t_k, p) > 0 \land W(p, t_{k+1}) > 0. \) Given a Mobile System MS, a subsystem resident in location \( L \) is isolated in marking \( M \) if there is no causal sequence \( t_1t_2...t_n \) with \( t_1 \in T_L \) and \( t_n \in T_{L_0} \).
Coloured mobile systems

Definition 20 (Consistent). A Coloured Mobile System MS is consistent if:

1. The colour set for a place is given by a tuple with the first element being a multiset of identifiers, the size being determined by the size of the relevant fusion set, i.e. \( \forall pf \in PF : \forall p \in pf : \theta(p) = \mu(ID) \times \ldots \land |\pi_1(\theta(p))| = |pf| \).
2. The colour set for a transition is given by a tuple with the first element being a multiset of identifiers, the size being determined by the size of the relevant fusion set, i.e. \( \forall t \in TF : \forall t \in tf : \theta(t) = \mu(ID) \times \ldots \land |\pi_1(\theta(t))| = |tf| \).
3. The firing mode of each transition shares an identifier with the consumed tokens, i.e. \( \forall p \in P : \forall t \in T : \forall c \in \theta(t) : \forall c' \in W(p,t)(c) : \pi_1(c) \cap \pi_1(c') \neq \emptyset \).
4. The firing mode of each transition shares an identifier with the generated tokens, i.e. \( \forall p \in P : \forall t \in T : \forall c \in \theta(t) : \forall c' \in W(t,p)(c) : \pi_1(c) \cap \pi_1(c') \neq \emptyset \).
5. Distinct subsystems have distinct identifiers, i.e. \( \forall M \in [M_0] : \forall CL_1, CL_2 \in loc(CL_0) : \forall p_1 \in LP \cap CL_1 : \forall p_2 \in LP \cap CL_2 : \forall c_1 \in M(p_1) : \forall c_2 \in M(p_2) : CL_1 \neq CL_2 \Rightarrow \pi_1(c_1) \cap \pi_2(c_2) = \emptyset \).
6. The consumed tokens provide all the identifiers found in the transition firing modes, i.e. \( \forall t \in T : \forall c_1, c_2 \in \theta(t) : (\forall p \in P : W(p,t)(c_1) = W(p,t)(c_2)) \Rightarrow c_1 = c_2 \).

- Colour makes things more descriptive but more messy formally
  - tried many alternatives for token and mode colours before finding the regular approach indicated

- Isolated subsystems can be determined in a manner akin to garbage collection
  - for a location to modify the root location, you need fusion
  - fusion requires that the context knows the identifier of the subsystem
  - this requirement implies that the subsystem is isolated but not vice versa